Teaching Geometric Locus using GeoGebra An experience with pre-service teachers

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Abstract: *We study the concept of locus using GeoGebra. Concerning the teaching of this concept, and the way it is being taught, there are important aspects to identify, such as meanings, definitions, visualizations and representations with a dynamic geometry system like GeoGebra. We present a case study on a group of pre-service teachers. The results allow us to identify how three visual, instrumental and discursive reasoning genesis are articulated in Geometric Work Space and to study the role which GeoGebra plays in the construction of Geometric locus.*

Keywords: geometric locus, pre-service teachers, visualization, visual reasoning, instrumental reasoning, GeoGebra.

1. Introduction

The concept of locus is as old as mathematics or, in particular, as geometry. In fact, humanity has always been fascinated by curves, even before they would see them as mathematical objects like in various prehistoric inscriptions. The Greeks started the study of different curves, from the simplest ones (lines, circles, conics) to more complex curves, obtained by studying the trajectories of points built from simpler curves. It is what we call today loci.

With the development of mathematics, mathematicians study more and more complex curves, and complex problems arise, often defined in terms of loci (Conchoid of Nicomedes, Deltoid of Steiner). Moreover, in the nineteenth century, many mechanisms, developed in Mechanics, can be defined in terms of loci. This allows us to know the real path of parts of these mechanisms, when, in the beginning, only partial approaches were known (Botana, Abánades, Escribano, 2010).

A geometrical locus is mathematically defined as the set of all points or lines that satisfy or are determined by *specific conditions*. Even this definition seems to be a clear one (or by quite natural way for solving), geometrical problems which proposed the search for a locus have been proved to be not so simple for the students to solve. Practically most of students encounter difficulties in demonstrating a geometrical locus and the constructions involved in the answer to the question "*Which is the path that follows this point if ...?*" are not so natural for regular students.

We consider that computerized learning environments open an avenue to the realization the potential of visual approaches and experimentation for mathematics to meaningful learning. In particular, the use of dynamic geometry systems for the concept of locus can help the teacher visualize classic constructions or curves in a dynamic way. It can lead the student to experiment and to find (at least partial) solutions to some problems as well. However, the tool itself does not provide this effectively. It is accompanied by operational ways in developing teaching systems.

Some researchers (Laborde, 2001, Gómez-Chacón and Kuzniak, 2010, Tapan, 2006) have indicated that, in general, undergraduate students (prospective teachers) lack the knowledge required to teach mathematics integrating educational software in the classroom. In fact, this problem is related to two different aspects. On one hand, the prospective Mathematics teachers are unaware of the development of mathematical notions in teaching situations. On the other hand, they have serious technical difficulties in using the software in a learning situation. These difficulties make it necessary to integrate specific work concerning the use of educational software into pre-service Mathematics teacher training

Motivated by this need, this article focuses on the study of the understanding of the concept of the locus by prospective teachers in dynamic geometry tools like GeoGebra. This aims to ask what kind of difficulties are inherent in teaching meanings, representation and visualizations that take place in a group of prospective secondary school teachers and how to take them into account in their teaching system in the Mathematics Department.

Using the computer, we can encourage visually-based concept formation in Geometry. We stress that the goal of the study is not only to provide solid visual intuitive support, but also to show the seeds for understanding formal subtleties that occur in the concept understanding of geometric loci when we use computerized environment. Several kinds of visual, analytic and instrumental thinking may need to be presented and integrated in order to construct rich understanding of mathematical concepts of geometric loci with GeoGebra. We consider that to foster this kind of versatile tool of mathematical reasoning for prospective teachers, it may prevent some of the weaker ones from successful problem solving and its teaching.

In this article, some initial answers to the following two questions, stated as a point of departure for our research, will be provided:

- What kind of mathematical thinking do students exhibit when they use dynamic software in their locus problem solving approaches?
- What competencies are required for prospective students in Math Education programs to teach geometrical locus with GeoGebra?

In a first approximation to answer these questions, we carefully analyzed the results of an empirical study, conducted with 30 prospective students at the Universidad Complutense de Madrid, indicating the existence of different student difficulties according to the level of competencies in each of the domains considered: visual, analytic and instrumental.

The rest of the paper is organized as follows. First, we describe the theoretical framework for our research. Then, the research methodology used, more specifically in the analyses conducted in this article, is presented. A section on the results of all the analyses is presented afterwards. This includes some answers to the questions formulated above. Finally, the first conclusions of our work and some suggestions for future studies are presented.

2. Theoretical Framework: Computers and Geometrical Locus

Much research has been carried out showing concern about the learning of Geometry (e.g. Cobo, Fortuny et al., 2007; Hershkowitz, Parzysz & van Dormolen, 1996; Jones, Gutiérrez and Mariotti, 2000, Mithalal, 2010; Richard et. al., 2007). Most of these studies on Dynamic

Geometry were carried out on the learning of students and on the design of learning scenarios. The studies on teacher training, such as those of De Villiers (2004); Gomez-Chacón & Kuzniak (2010); Tapan (2006) and Restrepo (2008), are a minority. And specially those are focus on the concept geometrical locus.

Several projects have used the potential of computers and exploited them in the development of software to achieve specific learning goals. A large part of research was focused on question in designing and implementing distinct research programs (Arcavi & Hadas, 2000; Arcavi, 2008; Guin, Ruthven and Trouche, 2005; Santos-Trigo, 2004). All these experts try to answer question such as what tools students need to use in order to develop and appreciate the relevance of searching for meaning or making sense of data and results during their mathematics learning and what extent does the use of technology help students construct problem representation to explore meaning and sense of mathematical relationships. These studies lead naturally to discuss some of the ways in which parts of the mathematics curriculum and classroom practice a student learning may differ from the traditional approach.

In our bibliographic reviews, we have found out that there exist very few studies that analyze, in a didactic level, the concept of geometric locus. Some recent works on computational geometry have reviewed current approaches in the generation of geometric loci using dynamic geometry systems, comparing computer algebra systems and symbolic dynamical objects (Botana, 2002). However, these studies do not develop the teaching complement that a professor should have in the educational system. Some authors have compared the visual (and sometimes misleading) solutions that are generated by dynamic geometry systems, with exact solutions can be obtained using symbolic computation tools (Escribano, Botana, Abánades, 2010). This problem (the approximated solution) affects all dynamic geometry systems, due to the numerical nature of the calculations they perform. GeoGebra team has been working on improving this aspect, inside the GSoC[1] project. However, to obtain accurate answers for the meantime, we must use external tools[2].

Hence, we must observe that, when we construct a locus with GeoGebra (or any dynamic geometry system), we obtain a numerical approximation. We must be awarded that what we obtain is just a partial picture and we can also miss some regions of the locus, or complete algebraic components. This is not a problem for the advanced mathematician, but can be a real problem for a student.

Focusing on this problem, this paper brings together a diverse set of interaction techniques. Furthermore, this paper categorizes and describes these techniques accordingly to foster a better understanding of concept of geometric locus. In this way, this paper aims to provide a preliminary framework to help teacher educator or designers of mathematical cognitive tools in their selection and analysis of different interaction techniques as well as to encourage the design of more innovative interactive mathematical tools.

If in an enounced problem, a multitude of points are defined and the requisite is to discover and demonstrate what the geometrical locus of those points is, then new aspects relating to the delineation, delimitation and definition of the figure as a locus are coming out.

Even there are no established general methodology, every geometrical locus problem that aims to determine the locus has to be analyzed with great attention. This fact makes the

1 http://www.geogebra.org/trac/wiki/Gsoc2010

2 http://nash.sip.ucm.es/LAD/LADucation4ggb/

identification of three categories of geometrical elements necessary in the following problems:

- Fixed elements (position, length, dimension, volume);
- Mobile elements (position, length, volume, variable points);
- Constant elements (length, dimension, volume);

In dynamic geometry environment, we can explore what the locus aspect is by just moving the defining point. In any system, just activating the “trace” of the defining point, we can guess, in many situations, what kind of curve our locus is. But, to identify the locus, in a precise way, we must use the “locus” tool in GeoGebra. To use this tool, our construction must be sound and correct.

Furthermore, our interest in the study is to look through the interaction of two views: *cognitive difficulties of prospective students and instrumental handling from the expert phase to teaching.* These views offer us a better understanding of the difficulties the concept “Geometric Locus” with a dynamic geometric system like GeoGebra. They also help us in analyzing the effect of understanding to the performances of prospective teachers.

Cognitive view difficulties include, among other things, the reasoning with concepts in visual setting, to attain flexible and competent translation back and forth between visual and analytic representations of the same situation, which is at the core of understanding a great part of mathematics. Under *instrumental handling from the expert*, we would include issues of teaching. Although GeoGebra is a quite intuitive tool, in order to use it to teach (or to learn) mathematics, we must know the precise use of some tools (for example, the “locus” tool) and some basic techniques (like the use segments to control variable distances) that we cannot expect that regular students will learn by themselves in general. Their analysis suggests that teaching implies a “didactical transposition” (Chevelard, 1985) which, briefly stated, means the transformation knowledge inexorably undergoes when it is adapted from its scientific/academic character to the knowledge as it is to be taught.

3. Methodological aspects

A group of 30 prospective students was chosen as the study group. The focus of this research is exploratory, descriptive and interpretative in character. These characteristics take part – and are specific - of the qualitative research (Glaser & Strauss 1967; Latorre, 1996). For data collection, we have used a questionnaire with problems and semi-structured interviews (video-recording). In the interview, some questions where posed in order to elicit their visual and analytical reasoning and the difficulties regarding the visualization and the instrumental management.

The questionnaire is composed of 6 non-routine problems about geometrical locus to be solved using GeoGebra. Most of the problems are posed in an analytical register (Table 1).

The problems require the solver a proposed chain of various steps of visual processing, technical, deductive and analytical in order to find the solution. In this article, we have selected the responses that best represent the example to be shown, especially when the aim of our study is qualitative and has a descriptive intention of the identified behaviors. It has also been selected to present here example of these problems in which the locus was not an immediate display in the student's mind, not with pencil and paper.

The results obtained from the questionnaire required a deeper investigation of the individuals into cognitive, instrumental aspects related to understanding the geometrical locus. In order to do this, semi-structured interviews were conducted with 9 individuals chosen to represent the group.

The analysis focused on identifying evidence of how analytic, visual and instrumental processes interact during problem solving and research initials issues raised in the introduction section above.

Table 1: The Questionnaire of Geometric Locus Problems

PROBLEM'S HEADINGS	**DESCRIPTION**
Problem 1: *Find the equation of the locus formed by the barycenter of a triangle ABC, where* *A = (0, 4), B = (4, 0)* *and C is a point in the circle* $X^2 + y^2 + 4x = 0$.	*Level: basic* *Geometric Locus:* The statement of the problem determines the steps of the construction. *Visual-analytic reasoning:* Using the locus tool, Geogebra produces a precise representation of the locus. The student can only check the locus visually. To obtain an algebraic answer, it is necessary to take 3 points on the locus and then the circle passing by the 3 points. We now obtain an algebraic equation. *Instrumental reasoning:* use of the basic tools "line", "point", "middle point", "circle", "point on object", "locus".
Problem 2: *We consider a variable line r, passing through the origin O. Consider the point P where the line r intersect the line Y=3. From the point A = (3,0) we draw the line AP, and the line perpendicular to AP, say s. Find the locus of intersection points Q between the lines r and s, when we move r.*	*Level: medium* *Geometric Locus:* In this problem, the difficulty is to state, in a correct way, the meaning of variable line. Once this is correctly defined, the rest of the construction is quite straightforward. The statement gives explicit instructions for the construction. *Visual-analytic reasoning:* Using the locus tool, Geogebra produces a precise representation of the locus. The student can only check the locus visually. To obtain an algebraic answer, it is necessary to take 5 points on the locus and then the conic passing by the 5 points. We now obtain an algebraic equation. *Instrumental reasoning:* The moving point must be on a defined geometric object (line, circle, segment), and it can't be a free point.

PROBLEM'S HEADINGS	DESCRIPTION
Problem 3: *Consider a triangle ABC and a point P. Let us draw the projections of P on the sides of triangles, Q1, Q2, Q3. Are Q1, Q2 and Q3 in a line? What is the locus of points P such that Q1, Q2 and Q3 are aligned?*	*Level: medium – advanced* *Geometric Locus:* We can't draw the locus using the "locus" tool in GeoGebra, because it is a non-parametric locus. There is no mover point. *Visual-analytic reasoning:* GeoGebra is an essential help for the visualization. Although, in this case, despite activating the trace, we can't draw the locus. In this case, the reasoning with paper and pencil is essential, and this reasoning can be helped by the visualization of the problem. *Instrumental reasoning:* The construction is straightforward using the basic Geogebra tools. We need to use the "perpendicular line" for projections.
Problem 4: *A 5-meter ladder is supported at its upper end by a vertical wall, and its lower end is on the ground. What is the locus described by the midpoint M of the ladder, when it slips and fall to the ground? And if we consider a point that is not in the middle of the ladder?*	*Level: medium – advanced* *Geometric locus:* The statement does not give explicit instructions for the construction. The situation is realistic and easy to understand, but the translation to a GeoGebra construction is not evident. We need to use an auxiliary object. *Visual-analytic reasoning:* Once we overcome the initial difficulty, drawing the stair using an auxiliary object, Geogebra produces a precise representation of the locus. To obtain an algebraic answer, it is necessary to take 5 points on the locus and then the conic passing by the 5 points. We now obtain an algebraic equation. *Instrumental reasoning:* There are two key moments in the problem: 1) The construction of the stair with an auxiliary circle, and 2) if we want to study the locus of the positions that a point in the stair describes, the point must be defined in a precise way (middle point, "1/4 point").
Problem 5: *A 5-meter ladder is supported at its upper end by a vertical wall, and its lower end is on the ground. What is the locus described by the midpoint M of the ladder, when it slips and fall to the ground? And if we consider a point that is not in the middle of the ladder?*	*Level: Advanced* *Geometric locus:* The problem is clear using paper and pencil. The difficult point is to translate de idea of "distance" to GeoGebra. *Visual-analytic reasoning:* We combine the analytic register, with paper and pencil, and the visual register with GeoGebra. *Instrumental reasoning:* It is necessary to specify the distances using an auxiliary segment. Once we have this segment to control the distances, the construction is straightforward. We must note that we can't use a slider.

PROBLEM'S HEADINGS	DESCRIPTION
Problem 6: *To find the equation of the locus of point P such that the sum of the distances to the axes equals the square of the distance to the origin. What geometric object is this locus?*	*Level: Advanced* *Geometric locus:* The problem is clear using paper and pencil. The difficult point is to translate de idea of "distance" to GeoGebra. *Visual-analytic reasoning:* We combine the analytic register, with paper and pencil, and the visual register with GeoGebra. *Instrumental reasoning:* It is necessary to specify the distances using an auxiliary segment. Once we have this segment to control the distances, the construction is straightforward. We must note that we can't use a slider. The main difference with problem 5 is that we must define a line with the input window.

4. Cognitive perspective: student's geometric constructions with GeoGebra

In this section, we describe the different typologies that appear in the solutions of the six problems, in the group of study:

Typology 1: Static constructions (discrete) In this typology, students use GeoGebra as an advanced blackboard, but they do not use dynamic properties. They repeat the constructions for a number of points. To draw the geometric locus, they use the tool "conic by 5 points". This kind of difficulty shows up in problems 1 and 4.

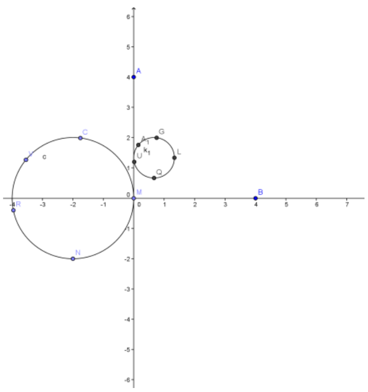

Figure 1. Student 6, solution problem 1

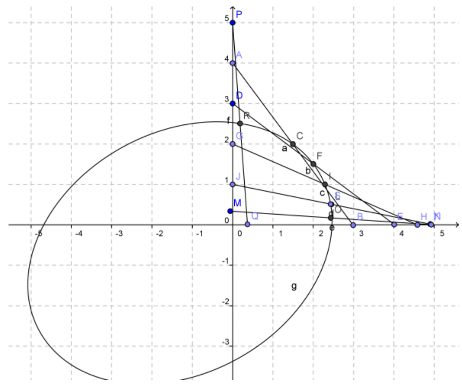

Figure 2. Student 13 , solution problem 4

Typology 2: Incorrect definition of the construction. The students solve the problem (in an imprecise way), but this kind of solution implies that the tools in GeoGebra cannot be used. To use the "locus" tool, it is necessary that the defining points are correctly determined (they can't be free points). With this approach, in the best

case, the students can obtain a partially valid construction, but, as we can't use the GeoGebra tools, we can't obtain an algebraic answer.
We find this typology in problems 2 and 4. In problem 2, the sheaf of lines should be defined by a point on an auxiliary object, as a line, and not by a free point. We obtain an approximate visual solution which is not usable with GeoGebra. The students in this typology are absolutely convinced that their solution is right. They are not at all aware that there exists a problem with the solution.

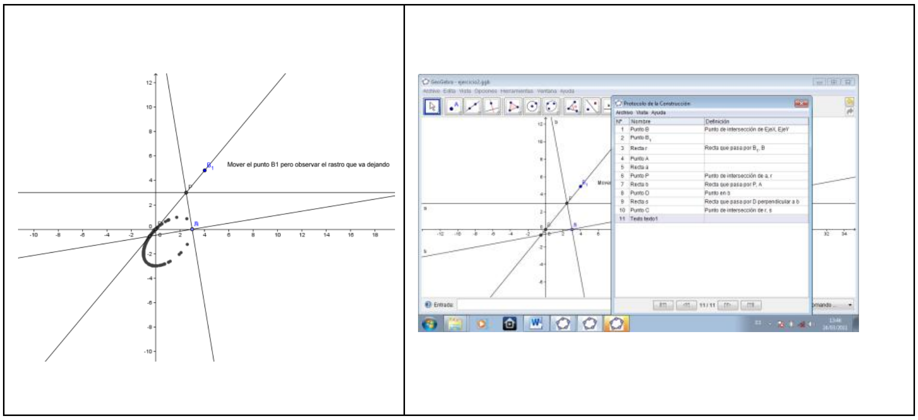

Figure 3. Student 6, solution problem 2

In problem 4, the difficulty is to define a point that is not the middle point. If we just take a free point on the ladder, we can't use the locus tool.

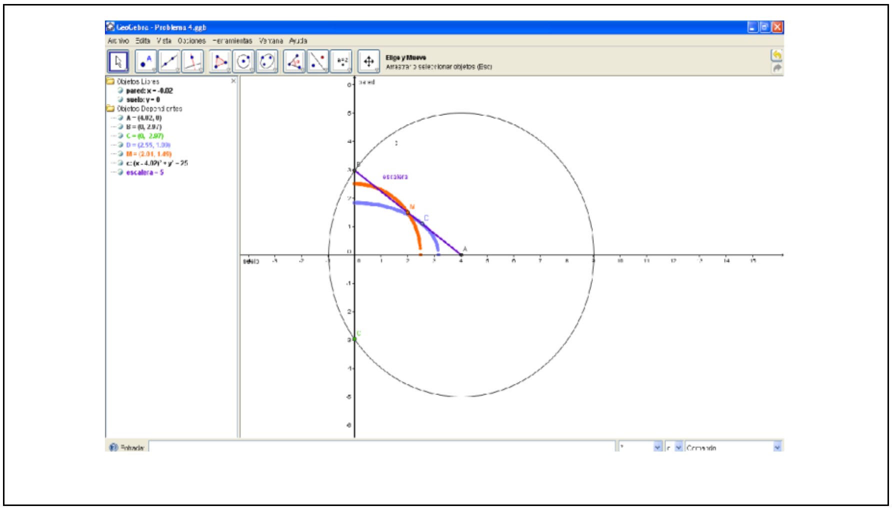

Figure 4: Student 23, solution problem 4

Typology 3: Invalid instrumental elements. For example, in problems 1, 2, 4 and 6 some students use the “slider” tool to displace the “mover point”. The student realizes that the “mover point” must be controlled, and the control is done by the slider. The problem is that, for GeoGebra, the slider is a scalar so it can't be used with the locus tool.[3]

As a representative case, we consider problem 2. Some students define the sheaf of lines as the lines passing through the origin on a point in the circle, and this point in the circle is moved with a slider. For example, student 9 says: “The problem is similar to the previous one. I have made the construction while reading the statement. The more complex step was to construct the variable line. First, I thought of a slider for the slope of the line passing through the origin, but, in this case, I will never obtain the vertical line, hence I used the slider as in the previous case to build a point C that turns around the origin, and then to build the line connecting C and O. From this moment, I just followed the instructions of the problem, and I took care of the names of the elements.” (Student 9, problem 2).

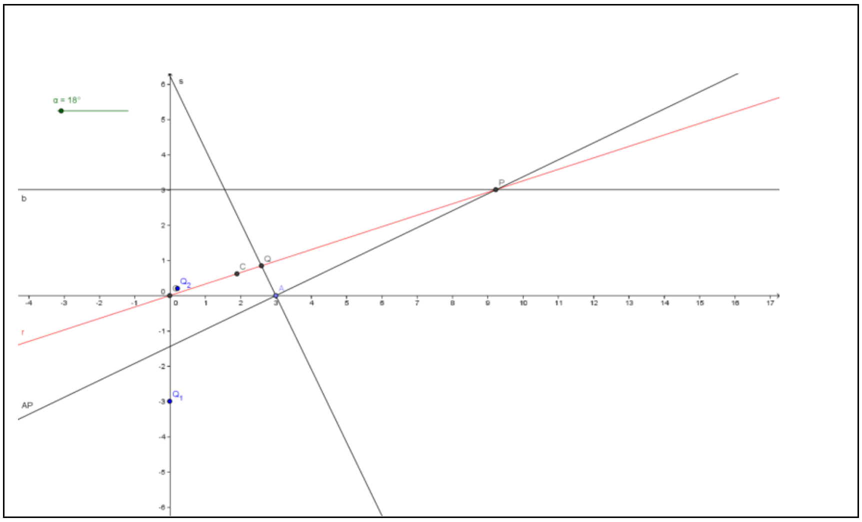

Figure 5: Student 9, solution problem 2

In problem 4, student 17 uses the slider to simulate the movement of the stair. She performs the necessary calculations with paper and pencil to determine the line that defines the segment of the stair. The line depends on a parameter that is controlled by the slider. The student says: “I have read and understood the problem. Next, I have drawn it in a paper, to visualize it better. I have calculated the points on axis X and axis Y where the stair is supported, such that the length equals 5 meters. (These points depend on a parameter A). The points are (0 , A) and (sqrt(25-A^2) , 0). Then, I have calculated the equation of the line passing by both point, and the equation is *x/ sqrt(25-A^2) = 1-y/A*. I drew the line with GeoGebra and used the slider for varying A between 0 and 5. Not knowing how to do to shift the line, this step made me curious, and a little anxious, wanting to resolve it as quickly as possible. After that, I drew the midpoint of the two previous points and then, I observed that the locus described by the latter point was an arc in the first quadrant (circle centered on

3 http://www.geogebra.org/help/docues/topics/746.html

(0, 0) and radius 2.5). To draw it in GeoGebra, I used the “circular arc” tool given its center and its ends.

Typology 4: Not using the locus tool. In this case, the construction is correct, but the student does not use the locus tool. To use the locus tool, we must clearly identify who is the point that trace the locus (tracer) and who is the point that moves the construction (mover). The mover must be a point on an object. We believe that there are some students who have no clear distinction between these points, and this situation leads them to the inability to use correctly the tool.

This type of difficulty is shown in problems 1, 2, 3 and 4. Let us select the student 8 to illustrate this type of reasoning: “The first thing I have to figure out is which is the center and radius of the circle to draw, completing the square in the equation $(x + 2)^2 + y^2 = 4$. Therefore, the point C is in the circle with center (-2,0) and radius 2. (Actually I do not need this because in Geogebra, I can directly enter the equation and I draw the circle). Now, we must know the concept of barycenter in order to develop the problem. We consider a point C on the circle (creating an angular slider angle for the point along the circle) and then we draw the triangle ABC. We calculate the barycenter of the triangle (I've drawn the medians in dashed green color, in order to facilitate to see that G is the barycenter of the triangle). Activating the trace of point G, and with animation, we see the locus. As we see the locus is a circle, we can now calculate the equation by finding three points G1, G2, G3 and activating the “circle through three points” tool. We enter the data in Geogebra: (x-0.66) 2 + y-1.34) 2 = 0.44 "(Student 8, Problem 1)

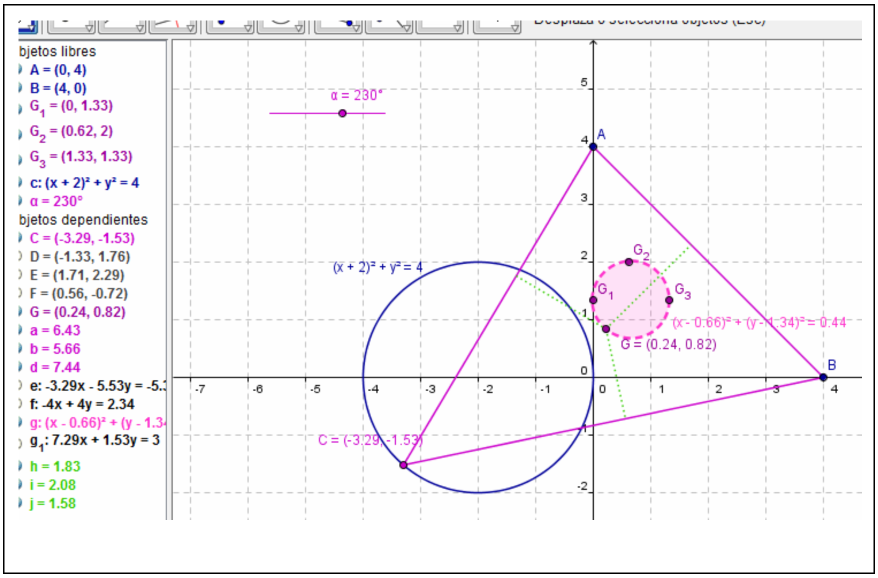

Figure 6: Student8, problem 1

5. Instrumental handling from the expert face to teaching

As described in the previous section on the typology of solutions generated by students was found behaviors that indicate a deficit in the interaction between analytic, visual and instrumental processes during problem solving. The problems proposed gives information about how students vary between intuitive and deductive reasoning when they have to solve a construction problem with software.

From a sociological perspective we must consider these issues facing education in the degree of mathematics and teacher education. Below is the solution of the expert, explaining how to connect the various arguments for better understanding of the concept. In these experts' solutions, we underline the importance of digital environment on relationships between the three geneses (figural, instrumental and discursive) of Geometric Work Space.

Problem 1:

Find the equation of the locus formed by the barycenter of a triangle ABC, where A = (0, 4), B = (4, 0) and C is a point in the circle $x^2 + y^2 + 4x = 0$.

It is easy to construct the circle with GeoGebra, completing the square: $(x+2)^2 + y^2 = 4$, that is, the circle of center (-2, 0) and radius 2. Or we can draw the circle directly, using the "input window".

Once we have our circle, the rest of the construction is easy, and we can use the locus tool to obtain an exact picture.

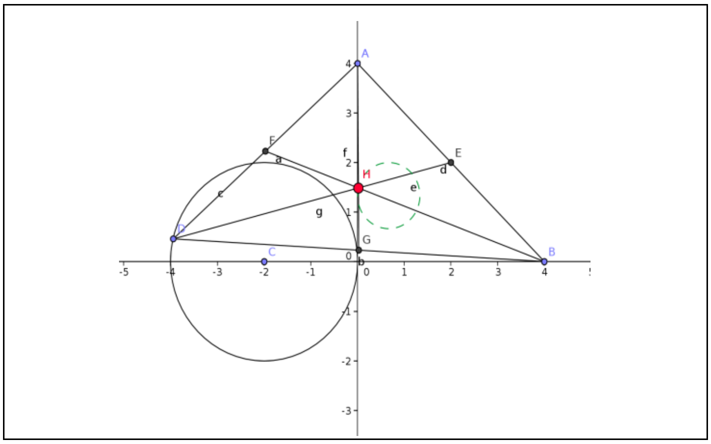

Figure 7: Solution Problem1

Problem 2

We consider a variable line r, passing through the origin O. Consider the point P where the line r intersect the line Y=3. From the point A = (3,0) we draw the line AP, and the line perpendicular to AP, say s. Find the locus of intersection points Q between the lines r and s, when we move r.

The problem is to clarify what "variable line" means. We can consider a sheaf of lines passing through the origin and an external point. In order to use the locus in GeoGebra, this external point must lie on an auxiliary object, a line or a circle. For example, we can consider the line y = -1, through the input window. We take a point on this line, and our family of lines will be the lines by and origin and the point. Once the concept of variable line has a precise meaning, the rest of the construction is straightforward, and with the locus tool we find the ellipse. Warning: We insist that it is fundamental to control the family of variable lines by a one-dimensional object (line, circle) in order to use the locus tool.

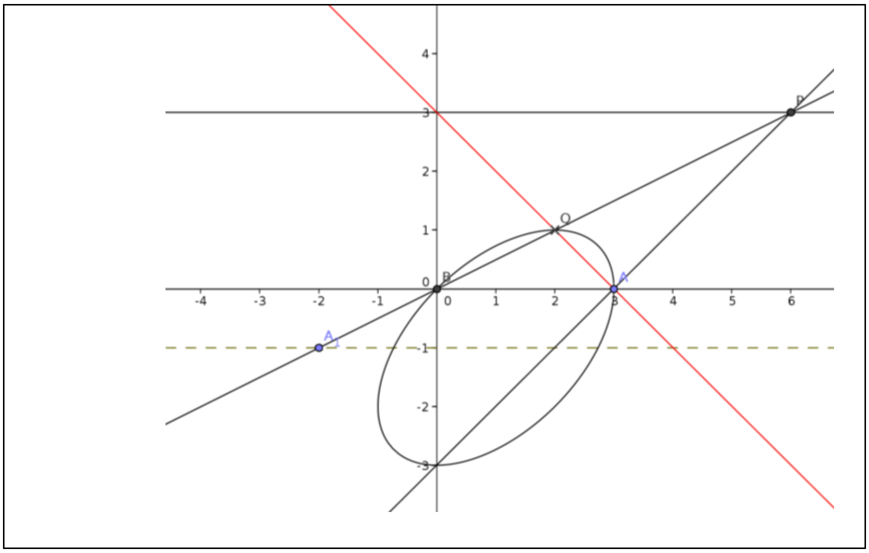

Figure 8: Solution Problem2

Problem 3

Consider a triangle ABC and a point P. Let us draw the projections of P on the sides of triangles, Q1, Q2, Q3. Are Q1, Q2 and Q3 in a line? What is the locus of points P such that Q1, Q2 and Q3 are aligned?

The construction is simple and the question is quite natural. However, the answer is not simple or obvious at all. This is an example of "nonparametric" locus and not solvable by GeoGebra (as of this writing). Conceptually, it is a different example of locus, which establishes a clear difference with the "parametric" examples.

Nevertheless, the students can guess the solution moving the point P, and considering an auxiliary line (passing by Q1 and Q2) to check if the three points are aligned.

Problem 4

A 5-meter ladder is supported at its upper end by a vertical wall, and its lower end is on the ground. What is the locus described by the midpoint M of the ladder, when it slips and fall to the ground?

And if we consider a point, that is not in the middle of the ladder?

In this problem, we can use GeoGebra to illustrate the situation, but, in general, we cannot expect that an average student will produce the full construction from scratch. The challenge is to manage with the sliding segment, which is achieved through an auxiliary circle (than can be hidden to produce the effect of the ladder).

In order to use the locus tool, we must to take care when we choose the point in the ladder. We cannot use just a "point in segment", we must use the "middle point" tool, or a more sophisticated construction.

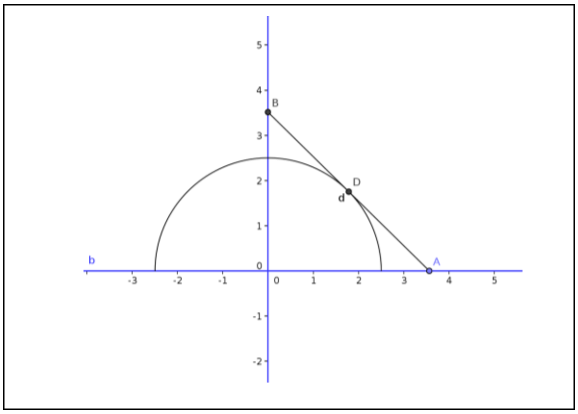

Figure 9: Solution Problem 4

Problem 5

Find the locus of points such that the ratio of their distances to the points A = (2, -3) and B = (3, -2) is 5/3. Which geometric object is?

The analytical solution of the problem is simple: Find the point P = (x, y) such that distance (P, A) = (5 / 3) * distance (P, B). Just pose the equations and, completing the square, we obtain the equation of a circle.

Using GeoGebra the solution is simple, but it may be necessary that the students have more advanced knowledge. To consider different distances (and to avoid problems with the locus tool), we use an auxiliary segment CD, a point E on the segment and the distance r between C and E. Now, we construct a circle with center A and radius r, and another circle with center B and radius (5/3)*r. The intersection of both circles generates two points in the locus. With the locus tool, we obtain a circle.

The key point is to consider the "variable distances", and to control it with a segment, and not, for example, with a slider. This can be difficult for some students.

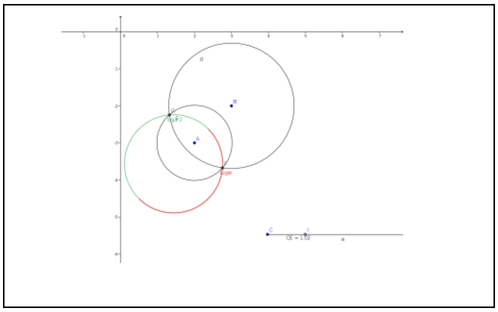

Figure 10: Solution Problem 5

Problem 6

To find the equation of the locus of point P such that the sum of the distances to the axes equals the square of the distance to the origin. What geometric object is this locus?

The problem is easy to solve analytically. It is enough to consider the corresponding equations:

$x + y = x^2 + y^2$, that is,

$(x - 1/2)^2 + (y - 1/2)^2 = ½$

Using GeoGebra, we must consider a more elaborated solution. We cover different distances *d* with an auxiliary segment (*d* is the distance from one end-point of the segment to a variable point in the segment) and we construct the circumference with center the origin and radius *d,* and the line $x + y = d^2$ (we can construct the line directly from the input window). We just intersect both objects and obtain the locus.

Attention: The "moving point" (the point that moves the point defining the locus, that is, the tracer) must be a point on an object (line, segment, circle). It cannot be a point on a slider.

Hence, it is an easy construction, but perhaps not very natural for regular students.

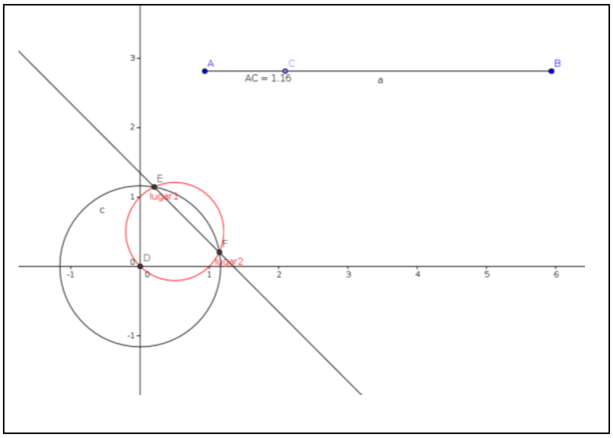

Figure 11: Solution Problem 6

6. Conclusion

One of the main goals of this research was to explore the understanding of the locus concept by prospective secondary school teacher in dynamic geometry tools like GeoGebra.

The data we collected show a wide diversity of approaches among students. This diversity comes from both their relationship with computers and software and also their relationship with Geometry. We identified four cognitive typologies of difficulties (section 4). These difficulties are related to both instrumental genesis and visual. This point is not surprising because some task emphasized the construction,

though students show a deficit of adaptation to instruments, especially of Locus tool. In cases that do not use the option command Locus (type 4), the construction is correct. However, in using the command locus, it must be clearly identified who the tracer of the locus is and what the point that moves the building is, that is, the point object (moving point). These students supported their reasoning on visual and discursive aspects. Returning to instrument to end the process can be problematic when there is no congruence between the theoretical tool and computer tool.
A second point to underscore is the difficulty (types 1 and 2) in this group of prospective teachers to distinguish between inert and interactive environment, by the capacity of the latter to respond physically to the "input" (Kaput, 1994).
We consider that GeoGebra is an essential help for the visualization, although with the problems proposed, we want to highlight the combination between visual and analytic registers is essential. There is a case in which we cannot draw the locus despite the activation of the trace. In this case, the reasoning with paper and pencil is essential, and this reasoning can be helped by the visualization of the problem. However, visual connotations of the concepts of locus on the computer go against a correct solution for a significant percentage of prospective students because they do not make a proof from an algebraic point. The challenge is to relate the visual, analytical and numerical dimensions of the subject so that they complement one another in the student’s understanding. In most interesting locus problems, there is a dynamic interplay between algebraic and geometric reasoning. The goal should be making the prospective teacher aware of this interplay and exploiting it creatively.
Finally, the character of computers or the availability of software packages with numerical, graphical and symbolic capabilities, does not guarantee that this integration will occur. The integration must be built into the structure of the course and into the design of particular topics and problems in teaching situation. In this study, some student behaviors have been identified. These can be used as examples in teacher training to especially especially show:

a) Schemes used to solve a geometric locus problem with software.

b) Schemes used to analyze and construct teaching situations with software.

In this perspective, it is necessary to introduce training modules that closely link the technical elements on the software to the elements of mathematics education in addition to the training situation of homology.

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