

Teaching Computer-Aided Design with GeoGebra

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Introduction

At most universities, Computer Aided Design (CAD) is instructed with commercial programs that are more suitable for business and industry than for teaching, since they have not been designed for this purpose. Most of the students finish the module knowing which buttons to push, not how to use the CAD software to enhance the design process and with very little knowledge on how CAD works.

Students are aware of the importance of mastering the commercial programs that they will encounter working in a technical office or in a shipyard in the case of our school, but a large percentage of them prefer to consolidate theoretical concepts and acquire practice with computer programs.

The objective of our school inside the Technical University of Madrid is to instruct Naval Architects, who will mainly work in a shipyard or in the shipping industry, where CAD plays an important role in ship design. For technical universities that instruct to future engineers that will be involved in different kind of industries, CAD is a common subject.

Technical drawing, geometry, and CAD are important subjects in the students' curriculum, and these subjects are taught in different modules throughout their years at university. This is common for all the engineering schools at Spain.

This paper refers to an intensification of CAD that along with teaching of CAD programs, embraces CAD foundations and algorithms. Students can select this subject in their 4th year of studies, when they have geometrical and mathematical basis to understand most of the algorithms and mathematics used in CAD programs.

These algorithms are extremely important because they are best suited to express data to a computer. This is because they are easy to program but not to easy to understand or to operate. This apprehension is improved if a visual tool or sketch is used.

Some important algorithms that provide the creation of Bezier, B-spline, and NURBS (Non Uniform Rational Basis Spline) curves are taught with the use of Geogebra software, since these algorithms have a graphical origin and application. Modern CAD programs use this family of curves to model solids and construct surfaces based on these curves as in Fig. 1.

A practical application of these numerical objects is that can be constructed with the use of numerical control machines that read the information of the surfaces, as in the bottom of Fig. 1 that shows a constructed prototype of a ship hull modelled mathematically with NURBS surfaces.

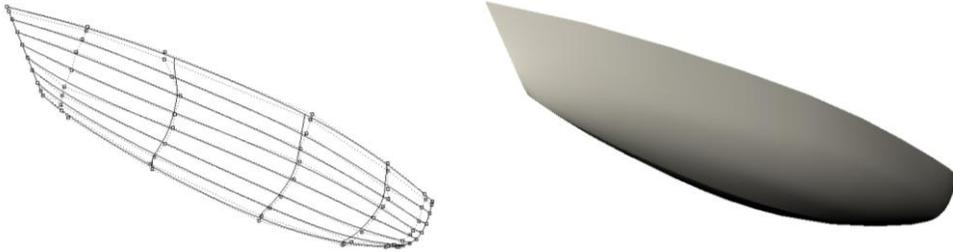


Fig. 1 Application of CAD to naval architecture

Although modern CAD programs can easily model ship hulls, the previous example is shown to the students to explain some surface fairing algorithms that are not implemented in the CAD program. With a good understanding of CAD basics, students can produce their own applications which can be very useful at a shipyard or technical offices.

This paper exhibits an experience of CAD teaching at university level, which demands some specific characteristics of the materials where GeoGebra is used together with CAD software. This methodology has been in use since 2007, and has been tested in different group of students through these years. Previously, only a CAD program was used for the course.

GeoGebra as a CAD support tool

Over the last decades, several visualization tools ranging from two and three dimensional CAD modelers, to systems for symbolic computations and visualization such as Maple (Mapplesoft 2009) and Mathematica (Wolfran 2009) have been developed. However none of these systems are well suited for direct manipulation of all the properties of the type of geometric figures and algorithms related with CAD. For example, three dimensional CAD systems are capable of creating and showing spatial objects, and show perspectives from any angle. CAD programs embody many geometrical and projective concepts, but have been designed to create objects that are visually appealing, not for mathematical study of their geometry. A CAD system has in general no deeper understanding of the underlying mathematical structure.

On the other hand, systems like Mathematica support symbolic computations and can simplify algebraic equations without recurring numerical methods. But these sort of programs are too slow and not as user interactive as a DGS (Dynamic Geometry Software) software, and also are not as intuitive or easy to use, as GeoGebra.

DGS programs appeared during the 80's and have been used specially in secondary education. At first tried to use computers as a compass and ruler replacement: the user can draw lines, circles, calculate intersections...using the mouse and keyboard, as can be made in a CAD program. But the great difference is that the computer records the way the user develop the construction and the software is able to quickly redo the construction after some parameter is changed.

The key of the interactivity is that these parameters can be changed with the mouse on the screen: grab a point, move it and see immediately how the construction changes. Programs for symbolic calculations require written command lines to produce the same operation.

A great application of the dynamic behaviour of a construction is that the students can see what parts of the construction change, and remain the same. This contains a central notion in geometry that is idea of the invariance. Another underlying advantage of working with a GeoGebra is that it is necessary to think geometrically because this provides the necessary intuition to make a construction. And this geometrical thinking is very important for an engineer since can be applied to other problems and disciplines as well.

The tool that the students use is central to the process by which students acquire knowledge and skills. Tools do not simply serve to facilitate mental processes that would otherwise exist, like geometrical thinking; rather they fundamentally shape and transform them, Cobb (1998). Hence, the geometrical conceiving is improved by the use of GeoGebra. Pioneering work by mathematics educators quickly associated dynamic geometry with a pedagogical orientation in which such software served to create experimental environments where collaborative learning and student exploration are encouraged, Balacheff & Kaput (1996).

Dynamic geometry systems were seen as pedagogic tools finely tuned for the exploration of a mathematical domain and as providing a setting in which students can construct and experiment with geometrical objects and relationships (Hoyles & Noss, 2003). The properties of the Splines can be intuited and experimented by the students better with the use of GeoGebra than with the use of CAD programs. In addition, GeoGebra programs do not include (yet) Splines, so students have to make the effort of use the appropriate algorithms to create this kind of curves from the scratch and obtain the instructional benefits of working this way.

Proponents of dynamic geometry have suggested its potential for changing the beliefs and the behaviour of professors in directions which reflect a changed view of mathematics, from a static, deductive activity that emphasizes proofs discovered by others to an exploratory, inductive activity that emphasizes the heuristics involved in discovering results (Ruthven et al. 2008). In the case of CAD, the properties of the algorithms that define the splines are more easily understood graphically with the use of GeoGebra, than mathematically.

Correspondingly, it has been suggested that dynamic geometry turns mathematics into a laboratory science so that mathematics becomes an investigation of interesting

phenomena, and the role of the mathematics students becomes that of the scientist (Gawlick, 2004), which is very important in a technical study like engineering. The use of DGS in Schools has been studied in some references; see Ruthven et al. (2008).

The use of a compass and ruler was the graphical origin of some CAD algorithms. Splines were used by naval architects in the XVII century to produce ships' master curves in order to construct the hull. A physical spline is a long, narrow strip of wood, plastic, or metal used to fit curves through specified data points, Rogers and Adams (1990). The splines are shaped by lead weights called "ducks" (Fig. 2, up) and by varying the number and position of the weights the spline is made to pass through the specified data, as smooth or fair as possible. These ducks and physical splines have a mathematical meaning (Fig. 2, down) and GeoGebra can be used to explain its properties together with a CAD program that students will use at work.

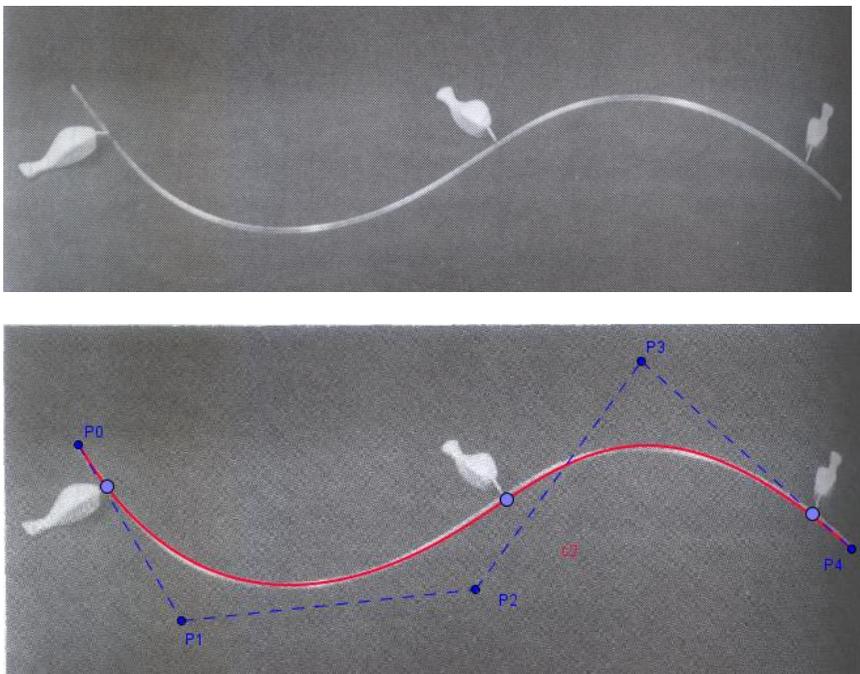


Fig. 2 The Loftman's spline and a mathematical B-Spline

Properties of spline curves can be varied with the use of a CAD program in a limited way since as mentioned these programs are more suitable for production and visualization than for teaching. With the development of dynamic geometry systems, dragging provided a more immediate method of continuous variation of a dynamic figure for similar purposes and with much more possibilities than in a CAD program. The use of automatic notation and the possibility of showing particular points with a mathematical meaning that are just

considered internally in CAD programs, are some of the examples that make GeoGebra especially appropriate to teach the algorithms and basis of computer aided design (Fig. 3).

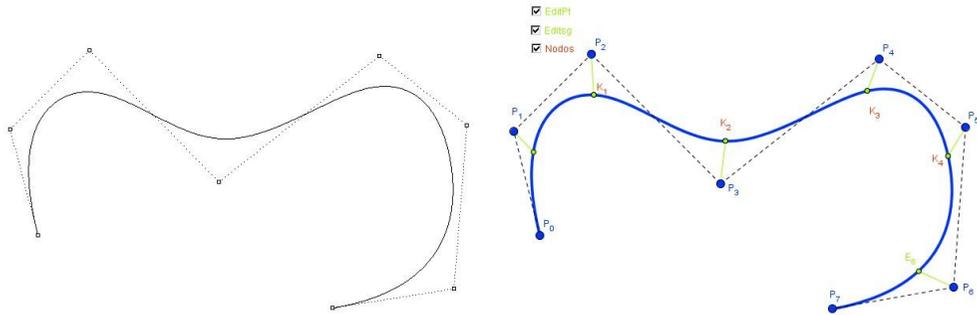


Fig. 3 CAD and GeoGebra presentation

The use of DGS demands more time on class preparation and on the time that the students have to dedicate to create the dynamic figures. This can be improved if some of the contents are offered as multimedia tools, which supplement the training that the students receive Culla et al.(2001). The teaching strategy would be very dynamic and colourful, increasing the attention of the students, Mathewson (1999). Just compare the CAD and GeoGebra presentation of Fig. 3; while the professor looks for the flexibility and potential of GeoGebra, students are more attracted to colours, motions, and nice representations.

Geometry at university level

Geometry and mathematics are very important to understand the basis and algorithms of CAD. A survey among CAD users in Xiuzi et al. (2004), shows that the role of an engineer involved with design is more than a mere CAD user, being a CAD application developer and a CAD software developer. The works of Xiuzi show that geometry, mathematics, and programming are very important for the engineer and every one related with design aspects will have the same opinion.

This is especially true in a shipyard since every shipyard and ship are different, and specific applications related to CAD, be it for design or production, are developed by the staff. Ship design and construction is not a very big market for big CAD companies, and commercial applications to solve specific problems at a shipyard are difficult to find. This can be applied to other engineering fields with specific fields: although commercial software is used, certain aspects related with production can demand home made applications of a user that knows well the production process and machine operation.

Geometry is intrinsically related with CAD. The time dedicated to Geometry in university studies has been reduced during the last years and in many cases, this is something difficult to change for a department. However, this can be improved with the use of DGS that permits the creation of interactive web contents, making geometry classes shorter and

more profitable for the students, and demands less time to study than with classic text book and blackboard lectures. This has been the author's experience for the last years of teaching geometry and technical drawing with the use of DGS.

Java Scripts web based contents and interactive multimedia technologies can be used to better communicate geometry and engineering concepts to the students. Learning with DGS, seems to be an attractive and effective way to develop spatial abilities. Nowadays, these technologies can be easily implemented and disseminated using Internet, thus providing resources to students with weaknesses in these fields. Quoting Paul Bézier, the inventor of Bézier curves, in the introduction of Farin (2002): "A short sketch is better than a long report"; with today technology the sketch can be dynamic.

Although the potential of DGS to create attractive web contents is clear, it would not be self sufficient for the students. A guided learning by the teacher is necessary especially in a technical subject like CAD or a goal of education will be lost (Kobal 2009). Attract attention does not mean necessary motivation, since motivation starts pushing the students towards acquiring new knowledge and deepening the old one, and this is done by the teacher that can be helped by effective tools as DGS programs.

Geometry is also basic to understand the CAD foundations. In the studies of Naval Architecture the student learns Euclidean geometry in the first course, and projective and descriptive geometry in the second one, apart from a CAD module, solely based on work with a CAD program without going into why it works. By having this base, the step to CAD basics and algorithms is easy and more effective when combining a commercial CAD program and DGS because the students will use the DGS software more intuitively due to their geometric and programming background.

DGS software

A number of DGS programs have been created over the last years and new software and updates are continuous. Just mention Cabri (Cabri 2009), Geometers Sketchpad (Sketchpad 2009), Cinderella (Cinderella 2009), GeoGebra (GeoGebra 2009),... All DGS software can be roughly divided in two categories: constraint-based and constructive. Constraint based software (Freixas et al. 2008) employs a general constraint method to restrict the position of objects, such as limit distances, or angles. In a constructive approach, objects and constraints are undividable. User can trace "tangent to an object" or "midpoint of a segment" that are both an object and a constraint.

The mentioned DGS programs are all constructive: they encourage the user to think about sequential use of Euclidean operations. This has been mentioned in section 2 as a useful technique to promote geometrical thinking. One problem of the constructive programs is ambiguities in the definition of some elements that will be mentioned in section 7. Constraint-based systems can be harder to manage but are more powerful from the design

point of view. For this reasons, constructive systems are more appropriate for teaching than constrained-based ones. Among the mentioned DGS programs, Geogebra was selected. Using Geogebra, the CAD algorithms can be used both graphically, that is essential for a good understanding, and numerically with the use of algebraic expressions previously obtained by the students with the use of the algorithms. The rest of DGS software mentioned only allows a compass and ruler definition of the elements, or at least they did not permitted this type of algebraically definition of elements in 2007 when the author started to introduce DGS into the subject mentioned in this paper.

An example is depicted in Fig. 4. On the left side of this figure, the Casteljau algorithm (Eq.(1)) is applied graphically to a cubic Bézier curve. In this way, students understand how it works and different points of the curve can be obtained by applying the algorithm repetitively.

Students starts with the control points $P_{0i} = P_i$, and then apply the algorithm to calculate the first stage P_{1j} and again to obtain points P_{2j} and for this particular cubic example, the final level of the algorithm produce P_{31} that is a point of the Bézier curve. A look on equation 1 shows that the curve will be a function of the control points P_{0i} and of a parameter u .

The value of this parameter can be varied and all the points P_{ij} will vary accordingly the algorithm producing different points of the curves, as depicted in Fig. 4 left. The variation can be made visually assigning a slider to the value of the parameter u that can be moved with the mouse, instead of typing a numerical value with the keyboard. This is especially attractive for the students because it is faster.

This way, a discrete number of points of the Bézier curve are represented with this compass and ruler construction. By dragging the control points P_i , the dynamic figure will clarify some properties related with the tangent values at the ends of the curve, but this discrete construction requires the “refresh” of the screen in order to clean the red dots representing the curve corresponding to the previous position of the control points.

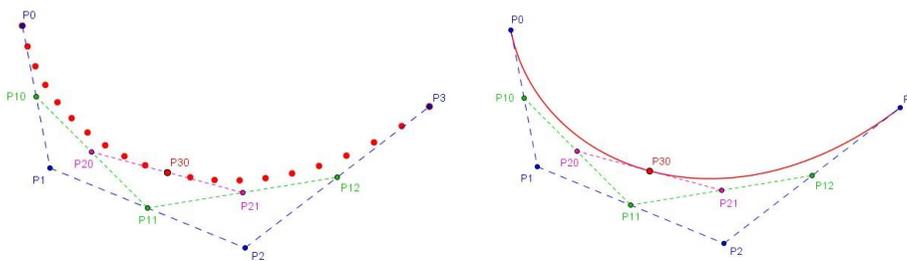


Fig. 4 Graphical and mathematical representation of a Bézier Curve of 3rd degree

On the right side of this figure, the same Bézier curve is constructed mathematically with use of the Cox de Boor algorithm, Eq (2). This curve is just a function of the control points P_i , but is compared with the graphical construction applying the algorithm.

Students have previously worked with the algorithm and obtained the basis functions analytically for the presented example. These basis functions are now introduced algebraically in Geogebra and a parametric curve can be constructed as shown in the right part of the figure. This way, students know how to work with the algorithm and obtain the mentioned basis functions for some prepared examples. This dynamic figure can be dragged without refreshing the screen since a continuous curve has been constructed. The properties of the Bézier curve are now clearer than with the previous graphical construction.

So, the same result has been obtained in two different ways, graphically and algebraically, with the use of two equivalent algorithms, using the capabilities of Geogebra. This is of paramount importance: algorithms are now more than equations as they have produced a dynamic construction that can be handled by the student dragging points and objects and changing values. The constructions are now more than a drawing since the student has considered the mathematics implicit in the lines and curves that can not be created directly in Geogebra.

With the use of Geogebra, classes are now a kind of laboratory, where the CAD is taught experimentally and the students can test the constructions and check the most important properties of Bezier, B-Splines, and NURBS curves, prior the construction of surfaces with these lines and applying CAD to specific engineering problems, in this particular case the representation of the ship hull with the use of NURBS surfaces or other problems related to shipyard practice.

Examples

In this section, some examples of CAD contents where DGS is used for teaching are presented. Students construct most of them while others are presented by the teacher, especially the introductory and the most complex ones. Relevant aspects of the examples have been summarized in tables 1, 2 and 3.

The course is divided in conventional classes where theoretical contents are instructed, and laboratory classes where computers are used with both GeoGebra and CAD programs. This joint of passive (“Hmmm that’s interesting, this course provide a lot of good information”) and active (“What happens if I do this?”) learning that requires a participating use of the information, produce good results in a practical subject as CAD.

Passive does not mean ineffective. Just because the students do not have to do anything with the information at that point of the course, it doesn’t mean that the teacher can not use new ways of presenting the information, and dynamic constructions can be very powerful pieces of information since produce in the student a more active attitude when attending the classes. Active learning is great because it allows the students to immediately apply the information or understanding and get feedback.

Bezier Curves with GeoGebra

It is important that the students understand the mathematical origin and the reason why Bézier curves and B-splines are used, and how they work. The origin is based on projective geometry and properties of conic curves that have been studied in previous courses.

The students will remember these concepts and other ones related with projective geometry studied in their second course, and can apply GeoGebra to understand the concepts. This first contact of the students with GeoGebra also serves to show the potential and to see the main characteristics of these programs.

Once these concepts are remembered and understood, the concept of a Bézier curve of second degree is introduced (Fig. 5, left), which is based on projective properties of conics (Fig. 5, right). The method to draw a parabola (a Bezier curve of second degree) is taught to the students at their first year.

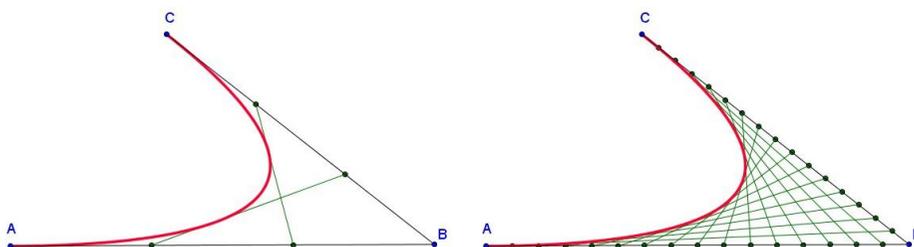


Fig. 5 Bézier curve of second degree (Parabola)

With this construction, that can be modified dragging the points A, B and C, the concept of control polygon of a Bezier curve and some properties related with the tangents at the ends of the curve are now introduced. The points considered as a Bezier curve can be calculated with an important theorem of CAD, the Casteljau algorithm:

$$P_{i,j} = (1-u) \cdot P_{i-1,j} + u \cdot P_{i-1,j+1} \quad \begin{cases} i = 1, 2, \dots, n \\ j = 0, 1, \dots, n-i \end{cases} \quad (1)$$

with $P_{0,j} = P_j$

Students use Geogebra to apply the algorithm in the case of a Bézier curve of third degree both graphically and numerically, using the Cox de Boor algorithm of Eq.(2) for the numerical solution.

The successful experience of constructing this figure tends to structure a later construction, which for this particular example is the application to Bézier curves of higher order. The graphical dynamic construction of the curves encourage a procedural effect focusing on the sequence of construction: this is the key to understand the algorithm.

Students naturally draw Bézier curves of degree 3 and 4 (Fig. 6) with Geogebra, both graphically and numerically. The students can move the control points of the curve and change the value of the parameter u in $[0, 1]$, and see how the different P_{ij} change

according to the curve is modified. This enables to see some aspects related with the derivative values of the curve.

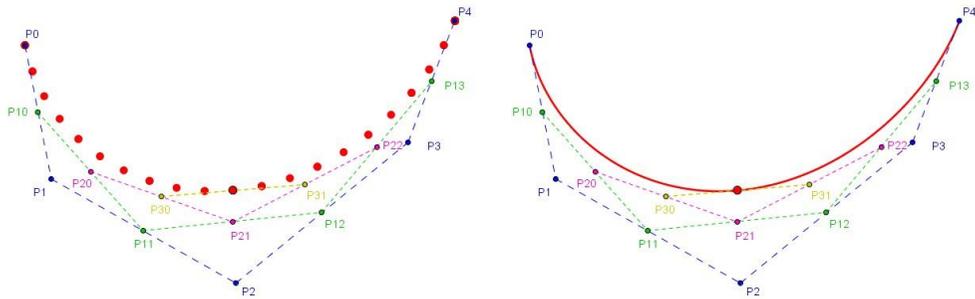


Fig. 6. Casteljau algorithm applied to 4th degree

Geogebra permits the creation of macros: once that a construction has been made dependant of some “input” objects, the “output” objects can be created automatically using a macro without repeating the intermediate steps. In this particular example, students produce different macros based on their own constructions of the Bézier curve. The input objects for a given degree will be the control points, and the output object will be the Bézier curve that has the input control points.

This is especially useful in our subject since these macros created by the students are used later in the course, and they create a kind of graphical library in a similar way as it is made in a programming environment. Students create their own library in Geogebra and do not use constructions made by other people that could be used as “black boxes”.

Once the students understand the algorithms below Bézier curves, both mathematically and graphically, they can practice with a CAD program that draws these kind of curves automatically. A logical extension from this point is the use of B-Splines.

Objectives:	Methodology:	Results:
<ul style="list-style-type: none"> - Study Bézier curves - Work with the Casteljau algorithm. - Work with the Cox de Boor algorithm - Draw Bézier curves - Show relevant properties 	<ul style="list-style-type: none"> -Students draw graphically Bézier curves of 2nd, 3rd and 4th degree in Geogebra, using the Casteljau algorithm. -Students deduct basis functions of the previous examples using the Cox de Boor algorithm and represent the curves - A CAD program is also used to represent the curves 	<ul style="list-style-type: none"> -Students analyze the properties of the Bézier curves with their own constructions. -Some properties can be easily to find, other ones are indicated by professor. - Some macros are created.

Table 1 Bézier curves

B-Spline Curves with GeoGebra

A B-spline curve of a given degree can be decomposed in Bezier curves, both graphically and numerically by repeated knot insertion (Piegl 1997). This technique is explained to the students, and they can program the algorithm that defines the equations of the curves. They can subdivide a B-Spline in Bezier curves, and draw them graphically as depicted in Fig. 7. Macros created in the previous example are used to create the different Bézier pieces that constitute the B-spline.

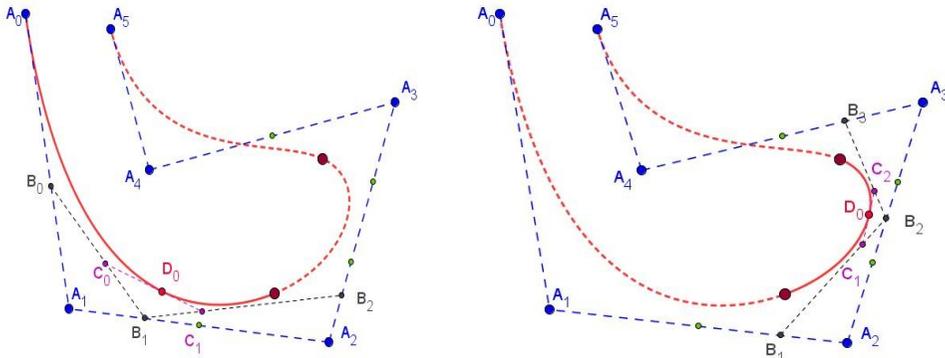


Fig. 7 Cubic B-Spline subdivided in Bezier curves

The same curve can also be programmed mathematically with the use of the de Boor algorithm, which demands the use of basis functions N of Eq. (2). The students have enough background to understand the properties of these basis functions and to program them numerically as they know how to use an engineering programming environment, such as Maple or C++. Some examples of low degree and number of control points are used to derive the basis functions, which can be represented algebraically in GeoGebra.

$$N_j^0(u) = \begin{cases} 1 & u \in [u_{j-1}, u_j] \\ 0 & u \notin [u_{j-1}, u_j] \end{cases}$$

$$N_j^n(u) = \frac{u - u_{j-1}}{u_{j+n-1} - u_{j-1}} \cdot N_j^{n-1}(u) + \frac{u_{j+n} - u}{u_{j+n} - u_j} \cdot N_{j+1}^{n-1}(u) \quad (2)$$

$$C(u) = \sum_{j=0}^n N_j^n(u) \cdot P_j$$

With the use of the examples created by the students with different degree and number of control points, the most important properties of B-splines used in practical applications are explained. One of these properties is the convex hull property (Fig. 8): a B-spline is contained in the convex hull of its control polyline. Students can construct the examples and drag the control points to understand the properties.

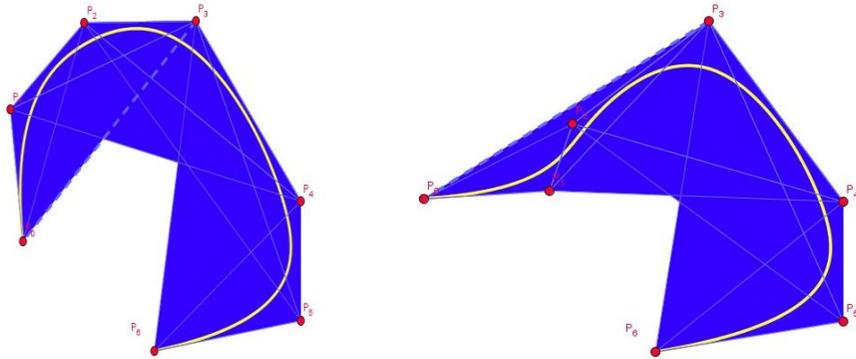


Fig. 8. Convex hull property

Another important property is the definition of the edit points: the displacement of a control point P_i has a maximum effect on an edit point E_i of the B-spline. This is significant because most naval applications use the modification of the control points in order to adapt the B-splines to the form of certain lines contained in the ship hull.

Students construct the B-spline, and will produce the edit points. With the use of GeoGebra, the path of a given point contained in the B-spline can be traced. This way the modification of a given control point, as P_2 of Fig. 9 that is moved in a circle c , produces similar circular variations in the points of the B-spline, with lower incidence as the point is far from the affected control point, and with parts of the curve remaining invariant. The highest effect corresponds to the edit point E_2 . CAD programs can not make a good visualization of these properties.

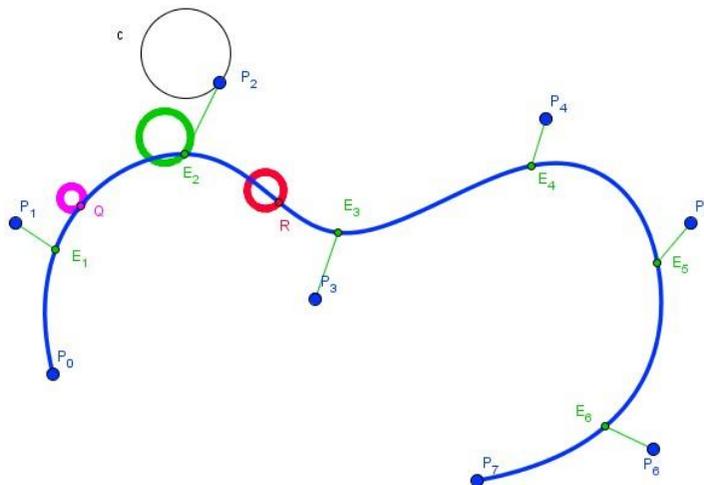


Fig. 9 Edit points: effect of moving a control point

Geogebra includes certain commands that are important when working with B-spline curves, for example the representation of the osculating circle. With this command, some concepts related with curve fairing are explained, as the osculating circle is directly related

with the concept of curvature, which is also related with the curve derivatives and with the selected degree of the B-spline.

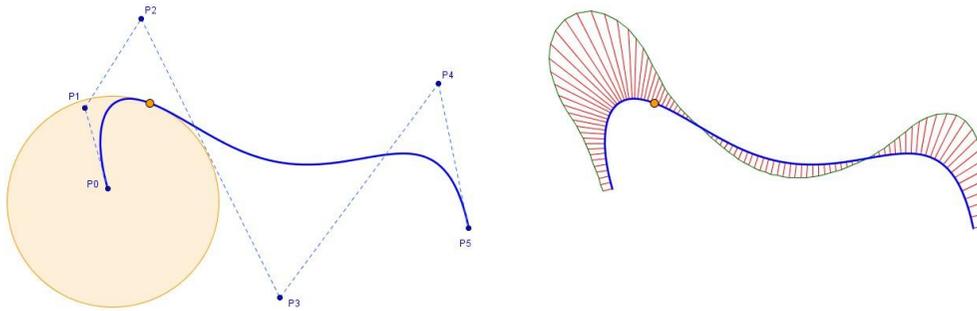


Fig. 10 Graphical and mathematical concept of curvature

In Fig. 10 left, the concept of osculating circle is applied to the properties of the B-spline curvature. The curvature plot of the B-spline can be represented with Geogebra as in Fig. 10 right. The osculating circle can be moved along the curve, so it is easy to understand the meaning of a curvature plot. Curvature plots are extensively used in CAD naval applications, as the ship hull lines have to be faired curves, and this is one of the practical applications that are included in the course. Some fairing algorithms for curves (Piegl 1997) are introduced with the previous dynamic constructions.

Objectives:

- Study B-spline curves
- Work with the Cox de Boor algorithm
- Work with the knot insertion algorithm
- Draw B-spline curves
- Show relevant properties and concepts

Methodology:

- Students draw graphically B-spline curves of 2nd, 3rd and 4th degree of low number of pieces in Geogebra, using knot insertion algorithm and Bézier macros
- Students deduct basis functions of the previous examples using the Cox de Boor algorithm and represent the curves
- A CAD program is also used to represent the curves

Results:

- Students analyze the properties of the B-spline
- Student represent specific points: knots and edit points.
- Relevant properties are studied: tangents with the created constructions: , deformation of the curve, fairing, convex hull property, derivatives,...

Table 2 B-Spline curves

NURBS curves

The natural extension once B-spline curves have been studied and grasped, are NURBS curves. A NURBS curve responds to Eq. (3) and is a function of certain weight w_i values.

$$C(u) = \frac{\sum_{j=0}^n w_j \cdot N_j^n(u) \cdot P_j}{\sum_{j=0}^n w_j \cdot N_j^n(u)} \quad (3)$$

This kind of curve can be constructed numerically with Geogebra for low degrees and number of control points, applying the basis functions previously studied and deduced with Eq. (2). Students program some examples and vary the weight and control points to see how the shape of the curve is altered (Fig. 11) when compared with the original B-spline curve of unit weights.

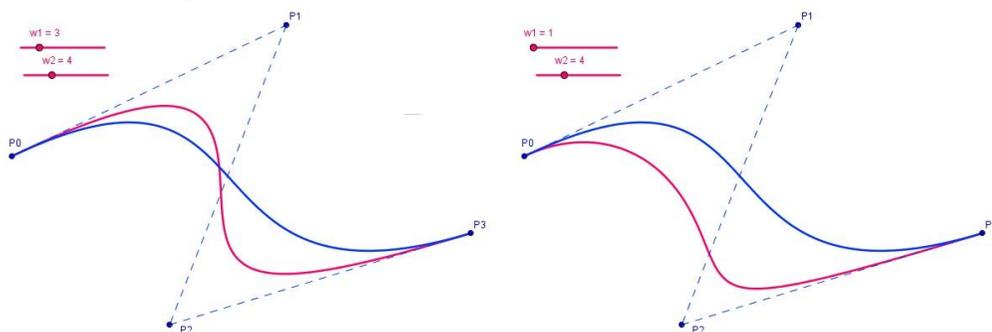


Fig. 11 Effect of weight value on NURBS curves

This example is based on the Bézier construction already generated by the students. As mentioned, earlier experiences of successfully constructing figures, enables to structure later constructions like this one, and it is positive to deep concepts.

Objectives:	Methodology:	Results:
<ul style="list-style-type: none"> - Study NURBS curves - Work with the Cox de Boor algorithm - Draw NURBS curves - Show relevant properties and concepts 	<ul style="list-style-type: none"> -Students represent numerically NURBS curves of 2nd, 3rd degree of low number of pieces in Geogebra -Students deduct basis functions of the previous examples using the Cox de Boor algorithm and represent the curves - A CAD program is also used to represent the curves 	<ul style="list-style-type: none"> -Students analyze the properties of the NURBS curves and compare with B-spline curves - Relevant properties are studied: tangents, deformation of the curve, weight effect,...

Table 3 NURBS curves

The mentioned examples are quite representative of the contents explained with GeoGebra. Until now, NURBS curves are well understood and the natural extension to

surfaces can be made. Students show a natural predisposition to this step and they easily understand the properties of NURBS surfaces, since they are a composition of NURBS curves.

Conclusions

When applied to Computer Aided Design, GeoGebra can be a very good option for teaching the algorithms and the basic underlying the graphical interfaces for CAD programs, such as Autocad, Microstation... In the case of an engineering education, the understanding of the basis and algorithms within the CAD program is important, so the engineer can create specific engineering applications depending on the field: aeronautical, naval, car engineering...

The results of the compared questionnaires show a reduction in the difficulty that the students perceive, and an increment on the comprehension of the algorithms and theorems that are used to create practical applications. Maybe the dynamic properties of the contents are the reason of the better understanding of the B-spline properties, since motion has a greater impact than a static figure or sketch.

Correspondingly, it has been suggested that dynamic geometry turns mathematics into a laboratory science so mathematics becomes an investigation of interesting phenomena, and the role of the mathematics student becomes that of a scientist, which is very significant in the case of practical knowledge as in engineering education. One of the disadvantages is that the use of GeoGebra demands more time in class preparation and in the time that the students have to dedicate to create the dynamic figures, which can be improved as they practice with the software.

Of course GeoGebra is not as automatic as CAD programs and the ambiguities that can happen in this program have been mentioned, but this is also positive since students have to think when making a dynamic figure, while a CAD construction can be made without much mental effort. This way, the use of CAD programs as “black box” programs, where the user does not know exactly what he or she is doing, and why it works, is avoided and geometric thinking is fomented.

According to the students' opinions, the algorithms are clearer with the use of GeoGebra, since they can “touch” the mathematics that they are programming and modify a lot of parameters, which are not common in CAD programs. Some talks with recent engineers that have developed CAD applications and were taught with DGS methodology, confirm that the understanding of the underlying algorithms and mathematics is essential.

With this background formed with GeoGebra and CAD knowledge, specific applications can be developed clearly, with a good understanding of the mathematics within the programmed algorithms. Author expects that the use of GeoGebra can be implemented by other instructors not only in their CAD subjects, but also in other courses with strong mathematical contents.

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