WITH SET SQUARES

SANDRA PANA, EMILIO SEOANE,
High school Profesor ,"Colegio Castroverde", Santander - España
sandrapana.pana@gmail.com

ABSTRACT: *In every isosceles triangle, the sum of the distances from any moving point E that lies on the base BC, to the congruent sides AC and AB, is a constant value. The difficulty of proving this property is its construction, due to the fact that is impossible to move the point E once is drawn on paper. Is that for, we will use GeoGebra, a dynamic geometry software, that allow us to move the point E, and then verify compliance with this property for any isosceles triangle and any position of the point E on the triangle base.*

PROBLEM OF STUDY

Create a GeoGebra file that allows us to observe that the sum of the distances from any point on the basis of the two congruent sides of the triangle is a constant value.

GEOGEBRA SOLUTION:

We need to build an isosceles triangle, by placing a point A and point B associated with a slider b, also placing a slider with its amplitude r, and generating two circles with centres A and B respectively. The point where the two circles intersect will be the vertex C of our isosceles triangle. Using the slider b, we can modify the coordinates of point B, thus we can obtain any isosceles triangle.

We place a point E that lies on the base of the triangle. From this point, we draw both perpendicular lines to the two sides AC and BC, so we get the segments EF and EG.

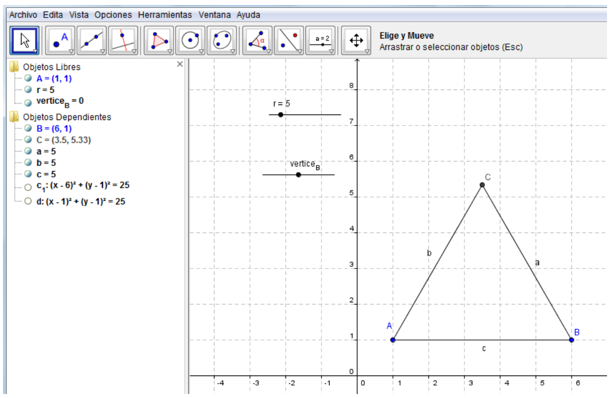


Fig 1.

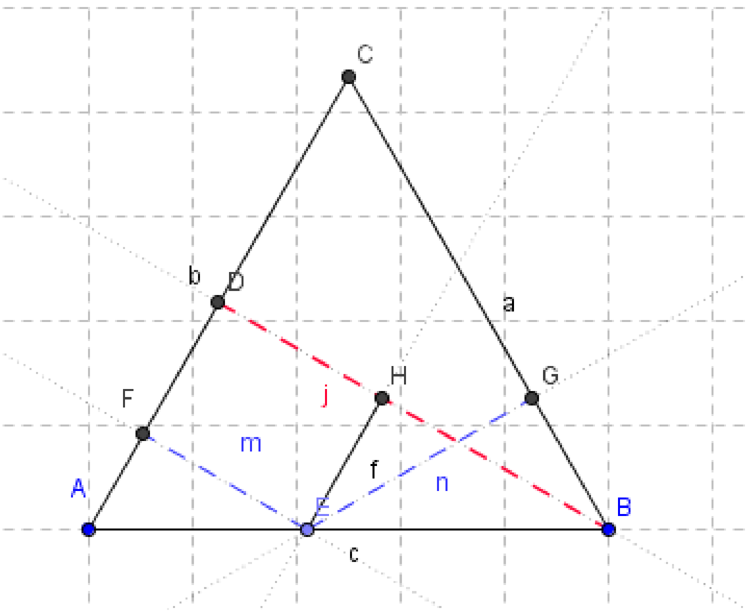


Fig. 2

We draw from point B a perpendicular line to the side AC (Fig. 2).
And also draw from the point E a perpendicular line to BD thus we get the EH segment. We slip point E along the base AB.

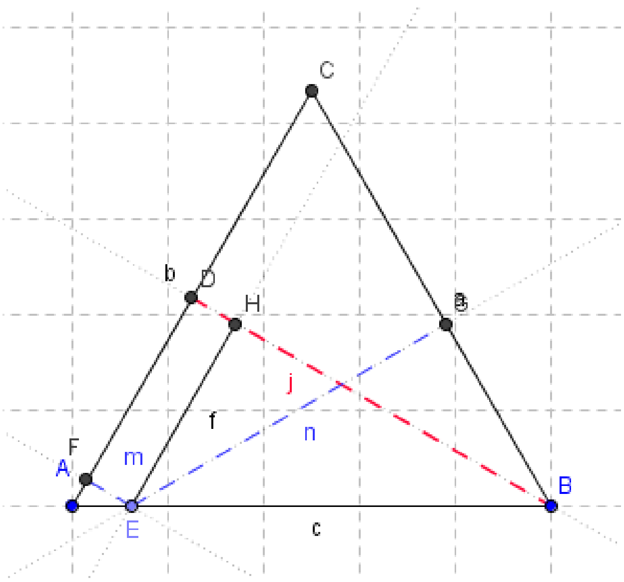


Fig.3

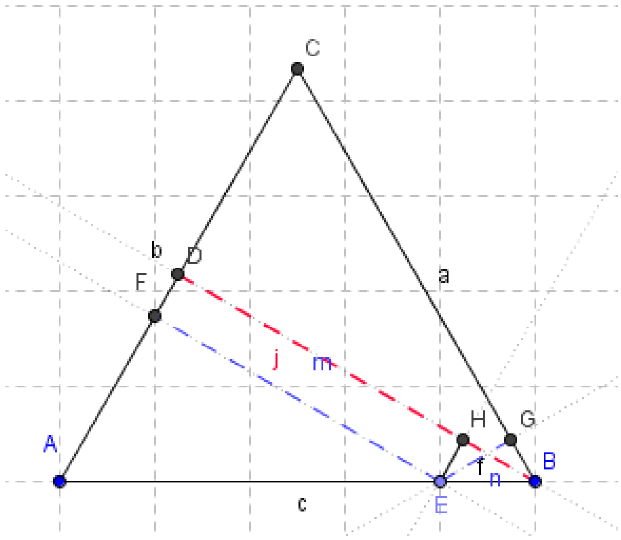


Fig.4

Once you create the segments AE (m in blue), EG (n blue) and DB (j in red) observe its dimensions. We also note that **m + n = j** for any position of point E. When E is at B, the length of segment **n** is zero and the length of segment **m** coincide with the length of segment **j**, so the sum of the distances from E to the congruent sides of isosceles triangle is constant value. (Length of segment j see figure .4)

MATHEMATICAL SOLUTION

As we have observed when E is at B, the sum of the distances to both congruent sides is equal to BD, because the distance to side AC is zero.
We prove therefore that:

$$EF + FG = BD$$

To do this we draw a parallel line to side AC that contains the point E. This way, we construct the EHDF rectangle, where EH = FD, and where we have to prove that EG = HB.
This is deducted from the equity △EHB=△EGB.
△EHB and △EGB are rectangled triangles at H and G respectively, and share a common hypotenuse EB. Acute angles HEC and GBE are equal, ∡ HEB = ∡ CAB which means congruent (parallel sides). Therefore EG = HB.
Therefore FE + EG = DH + HB = DB

Then the sum of the distances from E to the two congruent sides of an isosceles triangle is equal to the altitude drawn from one of the basic angles, distance therefore *constant*.

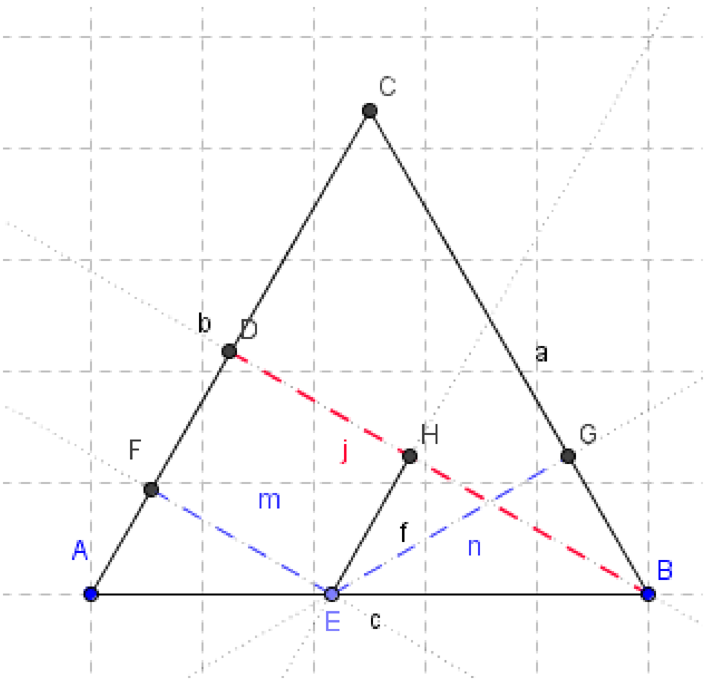


Fig.5

Subject knowledge:
An isosceles triangle is a triangle with two equal sides AC and BC (Fig. 6) and two equal angles (β = α) (Fig.7). Using both AE and BD altitudes to form two triangles △ABD and △AEB, we find that these triangles are equal by having a common hypotenuse (AB) and an equal acute angle and (β = α).

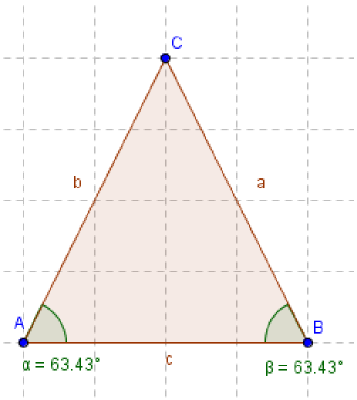


Fig. 6

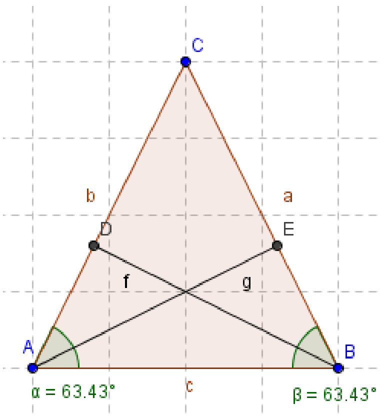
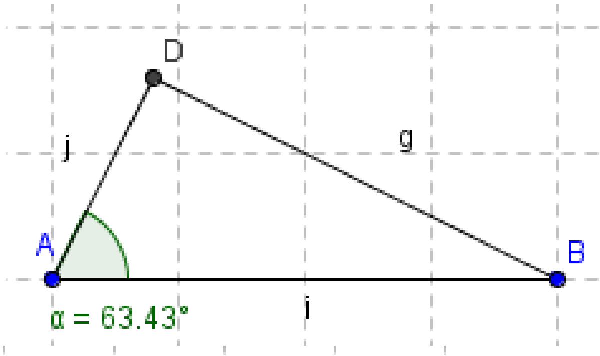
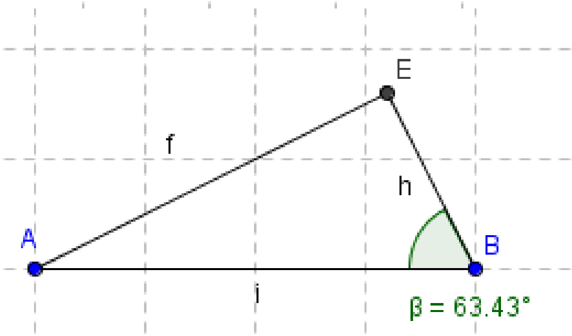


Fig. 7





Comments:

We have seen that the problem can be solved with set squares, but we are can not prove it at all points lied on the base. We have this chance by using GeoGebra, which dynamic feature can move the point E along the base. We can also see the construction step by step, which is a great advantage to understand the process and the problem.

Bibliography :

Curso superior GEOMETRIA Ediciones Bruño.