First Decade of GeoGebra: Looking back through Socio-Cognitive Lenses

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ABSTRACT. *In this chapter we situate and investigate the place of GeoGebra among the contemporary educational software systems in mathematics such as dynamic learning systems (DLS) and computer algebra systems (CAS). In addition, we explore the cohesive role of GeoGebra in setting up a virtual community of practice. By so doing, we simultaneously consider the social and cognitive roles of GeoGebra in the first decade since it has been launched. After addressing theory and practice of GeoGebra use in mathematics education, we discuss current trends in increasing teaching and learning potential of GeoGebra and suggest promising paths for research and development.*

1. Introduction

Technology use in education has been of interest for many mathematics educators and researchers since the very early days of technological innovations. Many still remember the first steps made with "an Apple II Plus (48K) computer and a CPM card" ([Hei03], p. 33) or some other primitive versions of hardware and software combination used to integrate technology in mathematics education in early 1980s. The very first attempts involved understanding computer technology and its capabilities, and exploring ways in which technology could be integrated ([KT08]) into mathematics education processes. From that point on, different roles have been assigned to computer technology, such as digitalizing mathematics content; developing software suitable for drill and practice; using software for simulation and exploration of mathematical models; creation of intelligent tutoring systems; and, in general, using technology to support cognitive activities, to extend cognitive abilities, and to (re)structure one's thinking, as suggested in the literature ([AT01]; [BV05]; [BGMU06]; [TB06]; [Pea85]). This process of educational development leads to the reasonable conclusion that the use of technology in mathematics education and the expectations of mathematics educators about technology have been evolving simultaneously and that the evolution is still in progress.

Through this evolution process, research in technology applications in mathematics education has been focusing on what technology to use and how (to be pedagogically meaningful) as well as taking into account specificities (e.g., language, representational forms) of mathematics disciplines, like geometry and algebra. As a result, different, more or less prominent, computer algebra systems (CAS) and dynamic geometry systems (DGS) have been developed, which further triggered discussions regarding the ways to integrate technology in mathematics education. The problem is that although some contemporary tools, or cognitive tools ([Pea85]), were specifically designed to enhance mathematics learning and teaching, the questions regarding the necessity of using technology in mathematics education still remain ([Gol03]; [Shu03]).

Goldenberg ([Gol03]), for example, discusses how using or not using technology in the domain of algebra may affect learning of "big ideas" in algebra. The author proposes that a "technological alternative to traditional algebra," such as "the intermediate value theorem and the fundamental theorem of calculus," may inspire new ways to help students understand the important ideas of mathematics (p. 24). Goldenberg acknowledges that "algebra is more than a language" (p. 25), thus recognizing the role of computer technology as an important tool for acquiring mathematical (algebraic) knowledge. In contrast, Kaput ([Kap92]), and Hoyles and Noss ([HL08]), describe the evolution of representational systems in mathematics from symbolic to visual. The "algebra as a language" is a useful metaphor in their analysis. But, even with acceptance of this stance, it should be noted that language could be learned through different means. For example, one can set a parallel between constructing "a house of mathematics" ([Kut03][1], p. 61) and learning language, and between learning formal mathematics rules and learning the grammar of the same language. The fact is that in order to learn language one does not necessarily have to start by understanding grammar first. Why, then, should students be asked to start with formal mathematics rules before they start building their own model of a house of mathematics? While one can argue that even technology use involves learning some (computer) language and rules of use first, these are often oversimplified in comparison to the formal mathematics language, and allow for a much easier move towards more complex applications.

In this chapter we place GeoGebra among contemporary software for learning and teaching mathematics, such as CAS and DGS, or dynamic learning systems (DLS) in general, within the cognitive and social perspectives. We start our discussion by specifying our theoretical framework, continue with the cognitive and social aspects of GeoGebra, and finalize the chapter with an overview.

2. Theoretical Frameworks

To theoretically situate our analysis, we follow Kaplan and Tripsas ([KT08]) who argue that it is to be expected that "cognitive processes should shape evolution" of technologies (p.). They emphasize that "when a technology first emerges, actors [in this case, users] are

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[1] Kutzler ([Kut03]) compares teaching and learning mathematics with building a 'house of mathematics,' which consists of stories. When building a house, each storey depends on the existence and sturdiness of the previous ones. Similarly, any mathematical activity consists of an exchange between lower level (old) and higher level (new) skills. If a student is not well versed in the old skills on which s/he needs to build the new ones, the student cannot progress further. Computer algebra systems can help here as a 'scaffold' which can enable a student to build a storey on top of an incomplete one.

unsure about what it is or how it will perform" (p.). Therefore, in order to make sense of the situational factors that provide more (in case of success) or less (in case of failure) fertile ground for some technology, cognitive explanations should be central to understanding technological evolution—a process that has thus far not received sufficient attention. By adopting this position as our framework, in order to look into the evolution of mathematics software GeoGebra, we apply *cognitive lenses* to understand its developmental trajectory and its present place among contemporary educational systems.

Here we return to Kutzler ([Kut03]) who suggests an inspiring metaphor to argue for the necessity of technology in mathematics education. The author sets an analogy between transportation, moving in general, and doing mathematics (see Table 1). This metaphor describes both curricular and individual progression through the stages in the mathematics learner's development, starting from mental calculations, and going through symbolic manipulations and technology applications, to provide more speed, accuracy, and scaffolds to those in need.

Table 1. Analogy between transportation and mathematics (based on [Kut03], p. 56)

Transportation and moving	Mathematics
Walking	Mental calculation
Riding a bicycle	Paper-and-pencil calculation
Driving a car	Calculator and computer

Furthermore, mathematics educators, who work in the domain of educational technology, consider technology as a cognitive tool which supports cognitive activities and extends cognitive abilities ([BV05]; [Kap92]; [Pea85]; [Swe03]). In that view the preceding metaphor could be rephrased to involve the use of technology as vehicles for (a) outsourcing procedural computations (e.g., being driven in a bus, taxi) and/or (b) exploring the landscape of various mathematics disciplines (e.g., driving along unknown terrain).

2.1. Categories of mathematics education software

The literature on the use of computer technology in mathematics education can be divided into two main categories: publications that focus on the use of CAS, such as *Derive* and *Maple*, which perform numeric calculations and algebraic manipulations and allow users to focus on conceptual understanding "by speeding up the process"; and DLS, such as *GeoGebra*, *Cabri*, and *Geometer's Sketchpad*, which provide users with environments suitable to explore mathematical concepts and relationships between them.

Although both, "CAS and interactive geometry software enable students to experiment with most topics taught in mathematics" ([Kut03], p. 62), the intentions for creating these tools were not the same. CAS, on the one hand, offer easy ways to perform many challenging, if not impossible, mathematical computations, including numeric and symbolic tasks. DGS and DLS, on the other hand, were developed to address mainly educational needs related to different learning styles and understanding of mathematics. Both categories are elaborated on in the further text.

2.1.1. Computer Algebra Systems (CAS)

Computer algebra systems are cognitive tools that "perform a wide variety of the numeric, graphic, symbolic, and logical operations that form the core components of algebra" and effectively "deal with numbers, symbolic expressions, equations, inequalities, functions, vectors, and matrices" ([CFKMZ03], p. 1). One can consider CAS as tools that extend users' cognitive abilities by releasing them from time-consuming and error-prone calculations and letting the users focus on higher order thinking demands of tasks.

By dealing with procedural computations, CAS are expected to enhance the learning of mathematics. The literature suggests that these systems are effective in (a) execution of routine procedures, either symbolic or numeric; (b) rote tasks; and (c) challenging, if not impossible, procedural tasks ([Hei03]); and that by employing CAS, teachers have the chance to allocate their students' time to exploration and discussion about the mathematics behind these tasks. The intention is to remove barriers that might prevent students from grasping a sense of fundamental mathematical knowledge and to develop a better understanding of the relations between mathematical concepts.

However, some authors are still concerned about the black box characteristics of CAS that may lead students to miss "the structure that lies behind the computations" ([Gol03], p. 28). The problem is that "students need an understanding of how mathematics works, not merely how it applies" and that abstraction in mathematics is still important (p. 28). The term 'abstraction' is seen as a process in which students "vertically reorganize previously constructed mathematics into a new mathematical structure" ([HSD01], p. 202), which with the use of CAS may remain underdeveloped.

2.1.2. Dynamic Learning Systems (DLS)

What we mean by DLS here, is a learning system that engages students in learning a specific concept by exploring it dynamically and interactively. As for mathematics education, a DLS is an environment that allows users to learn mathematics, including algebra and geometry. In that sense, we consider DLS as an umbrella term encompassing DGS and providing users with opportunities to create mathematical objects, to manipulate these objects within minimal constraints, and to observe the change in their features in time. From this perspective, one can also consider that these systems are cognitive tools intended to support cognitive activities as well as to extend the cognitive abilities of users by allowing them to exploit multiple representations and exploratory features of the medium.

Consider a scenario of a tangent drawn to a function from a point lying on a function. Calculus books state that the slope of such a tangent line is positive when the point is moving anywhere along the increasing part of the function, but diminishes while the point approaches the local maximum of the function. After the point passes the local maximum, the slope of the tangent turns negative, as the function decreases. This well-known *procept*[2] is a quite challenging phenomenon to grasp for many students who are taking calculus courses. The concepts related to the specific scenario described here are

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[2] The procept is an amalgam of process and concept ([Tal91]).

slope, increasing and decreasing function, and local maximum; the process could be accounted for by the movement of the point. This scenario illustrates a dynamic change in the slope of the tangent line associated with the change in the position of the point. The term *dynamic* which is used here points out exactly where the challenge may be experienced. In a pencil and paper environment this situation is usually presented by sketching a graph of the function with several tangents to the function around the local maximum, but this presentation may not be clear enough to the student (see Figure 1).

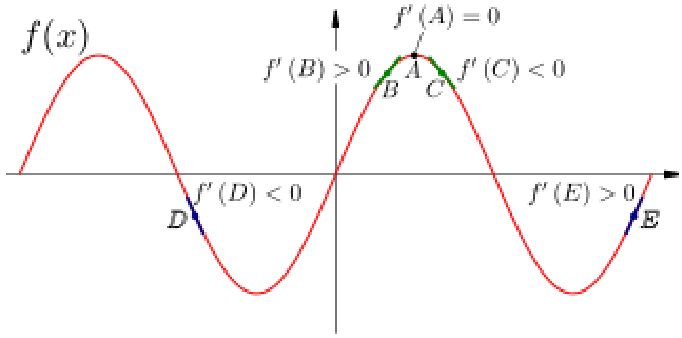


Figure 1. Presentation of change in sign of derivative/slope of the tangent (http://en.wikibooks.org/wiki/Calculus/Extrema_and_Points_of_Inflection)

However, DLS were intentionally developed to allow students to create dynamic worksheets to explore mathematical relationships similar to the one described in the scenario above. DLS extend the student's cognitive affordances by superimposing multiple static screenshots of the change in the position of a tangent and providing an animated simulation of the motion of the point along the curve.

According to Moreno-Armella, Hegedus, and Kaput ([MAHK08]) this feature places DLS currently at the top of a historical progression of mathematics representations. The authors claim that representational systems used in mathematics and mathematics education have evolved from a *static inert stage* to a *continuous dynamic stage*, and suggest that educational technology must be aligned with the needs of the stage we are in (see Table 2). In fact, the role of different representations and the importance of dynamic links between these distinct representations in mathematics education have been strongly emphasized in the literature ([Kap92]; [MAHK08]).

Table 2. The five evolutionary stages of representation in mathematics education, described by [MAHK08].

Static Inert	Static Kinesthetic / Aesthetic	Static Computational	Discrete Dynamic	Continuous Dynamic
Mathematics text is "fused" with the media it is presented upon or within (e.g., ink on paper).	This stage is characterized through the use of reusable and erasable media (in colour) (e.g., chalk and marker pens).	Characterized by a computational response to a human's action (e.g., a calculator with static representation of the user's input or interaction with the device).	The media is dynamic, as it is compliant with user actions, but re-animates notations and expressions on discrete inputs (e.g., spreadsheet).	Technology in this stage is sensitive to kinesthetic input (e.g., dragging of objects).

To sum up, while CAS also have animation features which could be used to demonstrate change in the phenomena, these can be achieved with a certain amount of programming skill, which usually relies on a teacher to develop. In the words of Lipeikiene and Lipeika ([LL06]), “CAS creates only opportunities. The problem remains for users to realize this potential” (p. 87).

It can be concluded that both CAS and DLS use different approaches to support students’ cognitive activities and extend their cognitive abilities. It seems that a combination of these two can greatly contribute to teaching and learning practices in mathematics education. More will be said about these two systems by using GeoGebra as an example.

3. GeoGebra: Towards balancing cognitive and social aspects of learning

Interactive mathematics learning software, GeoGebra, was created in 2002, by Markus Hohenwarter. The software was designed to support learners in building connections between two strands of mathematics, geometry and algebra, and, by doing so, to develop a deeper insight into the mathematical content ([Hoh02]).

In brief, GeoGebra allows users to create interactive mathematical objects, which are represented graphically, numerically, and symbolically, and to interact with these objects dynamically. Regardless of the user’s preference (because of the grade level, curriculum unit, or the learning style) for some of the three representations, the other representations are created automatically by the software and dynamically linked. In its present form (i.e., 3.2), GeoGebra offers graphics, algebra, and spreadsheet views that can be simultaneously utilized, or one-two at a time.

The software has been so far translated into 45 languages and can be either used in its online version (see http://www.geogebra.org/) or downloaded free of charge on the individual computer. GeoGebra went through several releases and received 12 national[3] and international awards[4]. Recently, there have been some ideas and attempts to use GeoGebra within an Intelligent Tutoring System (ITS, such as Geometry Tutor) ([MKC10]; [RFHG07]). A visual illustration of these developmental phases of GeoGebra is given in Figure 2.

In hindsight, it can be concluded that the first decade of GeoGebra has so far been marked by the mutual development of its social (Web 2.0) and cognitive (increasing options) aspects, which have contributed to strengthening GeoGebra’s impact on teaching and learning of mathematics.

[3] *Learnie* Award 2003/2005/2006—Austrian Educational Software Award; *digita* 2004—German Educational Software Award, etc.

[4] *Trophées du Libre* 2005—International Free Software Award, category Education; in 2009, the *Tech Museum* Education Award for innovative ways in using technologies for the benefit of humanity.

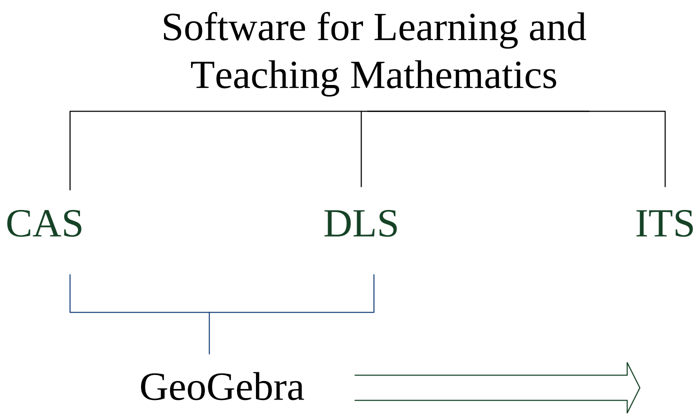


Figure 2. *Place of GeoGebra within the main categories of mathematics software systems*

In our further analysis, we will focus on GeoGebra and, by necessity, apply our "technological frames" ([KT08]; [OG94]) along with *social lenses* which guide our interpretation of what GeoGebra is and what it is useful for. Technological frames are important to define, as they are socially shaped by who people are — as users of the software, as educational researchers, practicing educators, and mathematicians. Developer(s) of GeoGebra, on the other hand, have their own technological frames, which are affected by their beliefs in relation to open source/open access models and the notion of communal knowledge building. In this analysis we also take into account the technological frames that some (especially government or school board) institutions have, because they too affect the directions that technology development will take. For example, government institutions prescribe school mathematics curricula; that is, they determine which and to what extent mathematics disciplines will be covered in schools and at what grade levels. This decision inevitably influences both the educational technology users and producers, as both groups, to some extent function within the boundaries set by the system.

To further clarify our position, we implement the framework of *socio-cognitive technological change*, which addresses both the social and cognitive factors that influence the evolution of GeoGebra. We note that socially, GeoGebra evolved from the product of one (i.e., Hohenwarter), was intended for use by some (because of its focus on geometry and algebra only; and for being dependent on command input), to the product of many (i.e., an increasing community of volunteers) and that it is intended for use by many (because of its increased scope, easy transfer to JavaScript, click-and-drag possibilities, open access). We also note that cognitively, GeoGebra in each of its releases increases the scope of mathematics curricula it is suitable for, as well as the number of representations it provides to its users. This feature is especially important because according to Kaput ([Kap92]), the use of linked multiple representations may be critical for understanding mathematics problems, or for developing a conceptual understanding of mathematics ideas in general.

3.1. GeoGebra as cognitive tool

In the study conducted by Hohenwarter, Hohenwarter, and Lavicza ([HHL08]), the authors gave four daily, 70 minute long GeoGebra workshops to 44 teachers, in an attempt to:

- Assess GeoGebra's usability and to identify challenging features and tools that could cause difficulties for novices;
- Establish complexity criteria and difficulty levels for GeoGebra; and
- Provide a basis for the improvement of professional development of secondary school teachers.

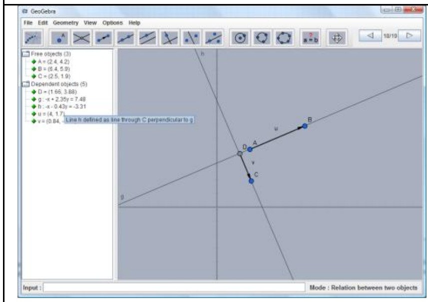
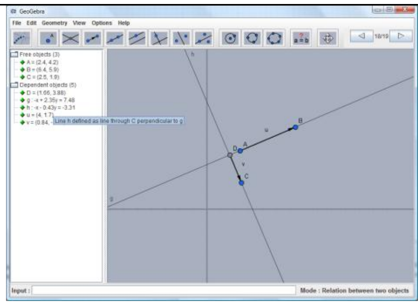
The results of this study highlight the ease of use and intuitiveness of the geometry side of the software as opposed to the steep learning curve required for its algebraic side. *The participants particularly needed assistance in geometric constructions and in proper use of appropriate algebraic syntax*. Based on this and other findings, Hohenwarter, Hohenwarter, and Lavicza adapted the workshops for teachers and developed detailed handouts to accompany workshop activities.

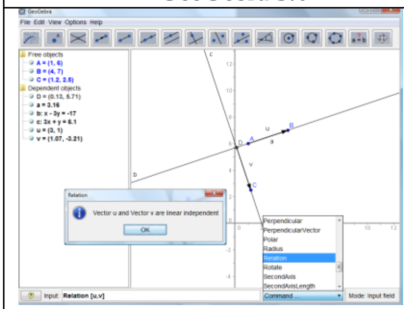
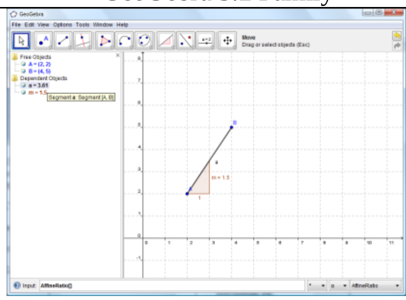
Other researchers have also investigated the interplay between Geometry and Algebra that exists in GeoGebra ([HJ07]). From that perspective, the basic objects in GeoGebra are not only points, vectors, segments, polygons, straight lines, and conics, but also functions in their implicit form. For example, a line g may be entered as g: $3x + 4y = 7$ or a circle c as c: $(x - 3)^2 + (y+ 2)^2 = 25$. This capability means that calculations with geometric objects, like points and vectors, are feasible.

Such a connection between two mathematics strands allows users to develop a skill in using algebraic formulas and calculations, while preserving the actual geometric meanings ([Ati01]). Along these lines, Giaquinto ([Gia07]) stipulates that presenting the relationship between geometry and algebra as a dichotomy is something of an over-simplification, since "the algebraic-geometric contrast, so far from being a dichotomy, represents something more like a spectrum" (p. 240). Banchoff ([Ban08]) notes that "Geometry and algebra are not just two subjects that appear throughout the curriculum; they are also distinct ways of thinking about mathematical ideas" (p. 99). Thus, students learn better when both ways of thinking are interwoven.

Since its first version (i.e., 1.0), GeoGebra has been significantly improving in both its form and capability (see Table 3).

Table 3. Differences in the feel and functionality between the first four releases of GeoGebra

GeoGebra 1.0	GeoGebra 2.0
 Release: January 28th, 2002 1. Available objects: point, vector, line, conics, angle, number 2. Has a "geometry look," besides the algebra panel 3. Constructions are done with the mouse 4. There is a limited selection of keyboard commands 5. Available in English, German	 Release: January 9th, 2004 1. Available objects: point, vector, angle, number, line, conics 2. Emphasis on geometry (including transformations) and analytic geometry (including conics and vectors) 3. Limited calculus functionality (derivative, integral) 4. Available 43 commands (including logic function "if") 5. Available in English, German

GeoGebra 3.0	GeoGebra 3.2 Family
 Release: March 23rd, 2009 1. The objects are organized by types (points, lines, polygons, circles, etc.) 2. Strengthened calculus functionality (e.g., area, lower sum, inflection point, root, polynomial, sequences and interpolation polynomials) 3. Many new commands (i.e. 42): Area, Curvature, Min, Mod, Locus, Semicircle, Taylor polynomial, etc. 4. Available with 85 commands 5. Available in 39 languages	 Release: January 24, 2010 1. Has the functionality of spreadsheet too 2. Capability for statistical data analysis and presentation (e.g., bar chart, box plot, quartiles, line of best fit, variance) 3. Works with determinants and matrices 4. Many new commands (i.e., 86): Binomial Coefficient, Insert, GCD, Reverse, Transpose) 5. Available with 171 commands 6. Available in 39 languages (not including versions)

3.2. GeoGebra as a socially cohesive tool

Today, GeoGebra is a shining example of a successful open access project, one which involves thousands of people worldwide who are working together to develop, share, and apply new technological and pedagogical ideas of how to use GeoGebra to enhance mathematics learning and teaching. To present transformation of GeoGebra from an individual, creative masterpiece to the emerging and fast growing worldwide educational community, we present miles*to*nes of its social acceptance and impact (see Figure 3).

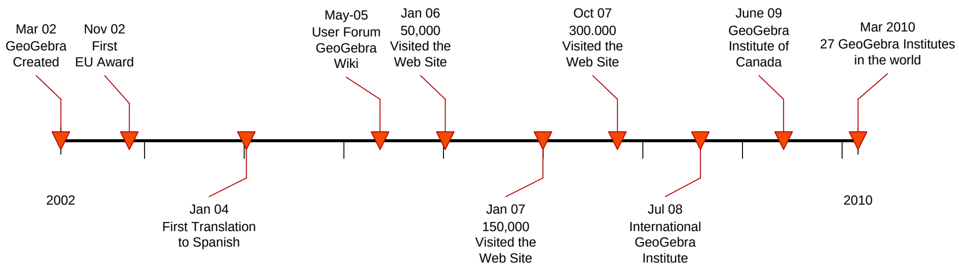


Figure 3. Selected milestones in change of GeoGebra's social impact

GeoGebra is an example of open source software for mathematics education at all levels. In Ljungberg ([Lju00]), the open source is described as a movement that is based on virtual networking on the Internet. This movement attracts a dedicated group of open-source software developers who are continuously improving and extending the software features. In many ways this community is connected to academia. Martinovic and Magliaro ([MM07]) state that one "strong commonality between these entities is a passion towards knowledge and treating it as a common good" (p. 47). However, what distinguishes GeoGebra from other commercial or free dynamic mathematics software is an ever expanding international community of mathematics educators, software developers, and researchers who follow the open source movement in software use and development, and carry out grassroots efforts in mathematics education reforms.

In essence, the GeoGebra community works on continually improving the software and on producing educational materials ([FSS09]). These distinct, but complementary features of GeoGebra are conceptualized in Figure 4 and are further elaborated on in the text.

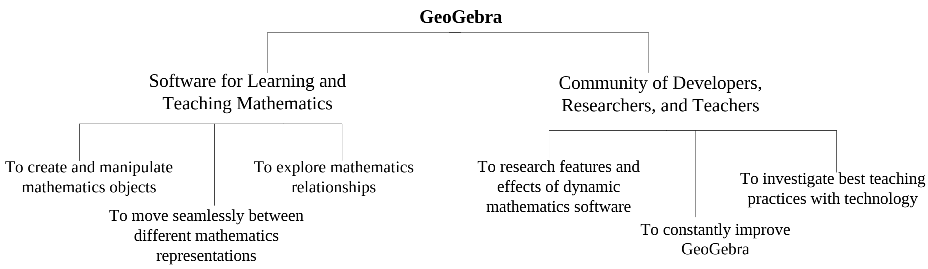


Figure 4. Conceptualization of GeoGebra as Software and as Community

Following modern educational trends towards enriching opportunities for teacher-student and student-student communication and collaboration, GeoGebra creates opportunities for teachers to collaborate by taking advantage of social interaction environments, such as *Teachertube.com*, *Facebook.com*, and wikis. However, this level of collaboration and communication goes far beyond what is expected at the K-12 levels. For example, teachers, academics and developers could exchange ideas and works-in-progress, rather than just dynamic mathematics worksheets. Given the accessibility of the World Wide Web, it is quite usual to meet someone from another country or even from another continent through the GeoGebra Web page on the Facebook and share success stories as well as challenges on some topics.

The GeoGebra community has been growing very fast for the very reason that GeoGebra is open source and free. Some researchers view open source as a sort of "gift economy" or gift culture whose raison d'être is the obligation to give, to receive, and to make a return for gifts received ([Mauss, 1950/1999, as cited in [Lju00]; [MM07]). Similar to other open source communities, in the GeoGebra community one's reputation among peers is the basis of competitive success. Such mutual support and motivation for providing assistance, which are characteristics of some online groups, are "partially founded on norms of generalized reciprocity and group citizenship" ([Wel97], p. 6). By using GeoGebra as "social glue" in bringing and keeping people together, the GeoGebra community demonstrates behaviours previously recorded among interactions emerging on the Internet ([Rhe98]).

The three kinds of collective goods that Rheingold envisions in such a community are *strength in numbers*, *knowledge pool*, and *cohesive empathy*, or, in other words, social network capital, knowledge capital, and communion. Other researchers ([EH05]; [ZP03]) also write about knowledge capital in the sense that people more strongly attached to online groups participate more and benefit more from their participation. By helping others in the group, the individual increases his or her social capital. By providing useful information to others, one increases their knowledge capital ([Rei98]). All three kinds of collective goods are present in the GeoGebra community. As a result, the GeoGebra Web site contains resources created by volunteers from all over the world. There is no surprise then that, as 2009 statistics show, GeoGebra is used in more than 190 countries and territories throughout the world.

Like other previously mentioned online groups, the GeoGebra community works on the concept of a tight core and loose membership. Open access to the software and the resources guarantees high numbers of visitors to the GeoGebra Web site, who in turn provide enough working and intellectual power for the whole group. Through time, knowledge grows when shared among community members; such forward development is visible in GeoGebra, as new ideas daily emerge in the community (e.g., "GeoGebra on a stick", GeoGebra in social networks, etc.).

In order to enhance communication among this growing community, the International GeoGebra Institute was established in 2008. This Institute coordinates communication and collaboration among local GeoGebra Institutes, which could be virtual, such as with online discussion groups and in online social environments, as well as through face-to-face encounters, such as at conferences and national or international meetings. One of the first activities organized by the International GeoGebra Institute was to bring local GeoGebra institutes and users together for the First International GeoGebra Conference, organized on July 14-15, 2009, in Linz, Austria and to brainstorm about the future of the community. Following this international gathering, many local GeoGebra meetings, such as the First North American GeoGebra Conference (July 27-28, at Ithaca College, NY) and the First Eurasia GeoGebra Meeting (May 11-13, Istanbul, Turkey) are already planned.

Establishing these activities and following up on them, is yet another unique aspect of the GeoGebra community—open access and open source GeoGebra constitutes not only an online community, as its influence and activities extend offline as well. By organizing workshops for teachers, new generations of users emerge. Their needs and ideas, this time coming from the school practice, affect further steps in GeoGebra's development. At each stage of this recursive process, new knowledge emerges from members' experiences, requests and recommendations, all of which will remain fundamental milestones for keeping GeoGebra alive, ever improving, and growing. These and other features of GeoGebra could be used in different forms of formal schooling. Furthermore, students could be invited to share their work online with their peers and a teacher in order to obtain feedback on their work. The innovations in the Web 2.0 (e.g., blog, wikis) technologies provide new opportunities for collaboration which could potentially let students develop beyond the parameters of individual work or work that involves only traditional means, such as books, paper-and-pencil, and talk-and-chalk. Therefore, the educational model of GeoGebra provides opportunities for an extension of classroom activities and for the removal of space and time restrictions of regular schooling.

4. Conclusions

In this chapter, we have reflected on the cognitive and social components in the emergence of GeoGebra from software to a vibrant and growing community of educators, developers and researchers. GeoGebra, as presented here, is a new model for the creation of communal and individual mathematical knowledge through interactive, dynamic, visual, investigative, and shared mathematical activities. From the *cognitive perspective*, mathematics is sometimes perceived by students as being 'disconnected' from real life and from other subjects (i.e., missing the trans-, inter- and intra- disciplinary links). As software suitable for learning and teaching of mathematics, GeoGebra was originally and genuinely planned as a 'linking' environment[5]. On the *social side*, the impact of GeoGebra may be important in shifting perceptions about mathematics as a solitary activity to the more

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[5] While GeoGebra started with algebra and geometry, now it links (nearly) all branches (of school) mathematics. From the present Ontario high school curriculum, only the Financial Mathematics strand is missing. All other strands are covered by GeoGebra: Number Sense and Algebra, Functions and Relations, Analytic Geometry, Measurement and Geometry, Trigonometry, Data Managements, Calculus, Algebra of vectors (see [OME05]; [OME07]).

harmonic view of mathematics as social activity ("if I can learn something new, discover it – I would naturally be happy to share it with someone, to discuss it, to argue for or against it"). This approach opens doors to communication and reasoning as socially active behaviors—not as absorption of already known facts and procedures.

GeoGebra as an open source learning tool also facilitates an interplay of social and cognitive aspects. Its users feel actively involved in the process of techno-pedagogical development, not as passive users, but as active creators of knowledge at all stages of the software development spiral—starting from an *idea* ➔ *design* ➔ *implementation* ➔ *assessment* ➔ *new ideas*, and so on. As such, GeoGebra provides for an inclusive environment as well as for socially meaningful and intellectually rich empowerment. This novel model will enhance conditions for paradigm shifts in teaching and learning as a co-constructive process.

GeoGebra as part of a research community provides a unique combination of virtual and in-person forms of collaboration (Web 2.0 tools and institutes) that leads to collection of evidence (stories), sharing and discussion of findings (cases), building of a common framework (shared understanding of phenomena), and construction of more empirical and longitudinal studies (as perspective), all of which are practice-grounded and practice-oriented, thus providing a strong feed-back system between research and practice.

In conclusion, we muse over the words of Jonassen, Howland, Marra, and Crismond ([JHMC08]), who stated that "A great deal of research on computers and other technologies has shown that they are no more effective at teaching students than teachers, but if we begin to think about technologies as learning tools that students learn *with,* not *from,* then the nature of student learning will change" (p. 6). By learning *with* interactive and dynamic mathematics software, such as GeoGebra, students may become *active* and *intentional* participants in the *authentic* and *constructive* process of meaningful learning in the sense described by Jonassen et al.

The words of Cutler ([Cut95]) may also be applied to GeoGebra as a community consisting of "persons [who] find new relationships worth cultivating, roles worth adopting, and selves worth becoming through activation of those roles" (p. 26). As such, GeoGebra presents a new education model which will have a lasting impact on both the future formal and informal schooling of mathematics students.

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