Methodological Considerations on the Use of GeoGebra in Teaching Geometry

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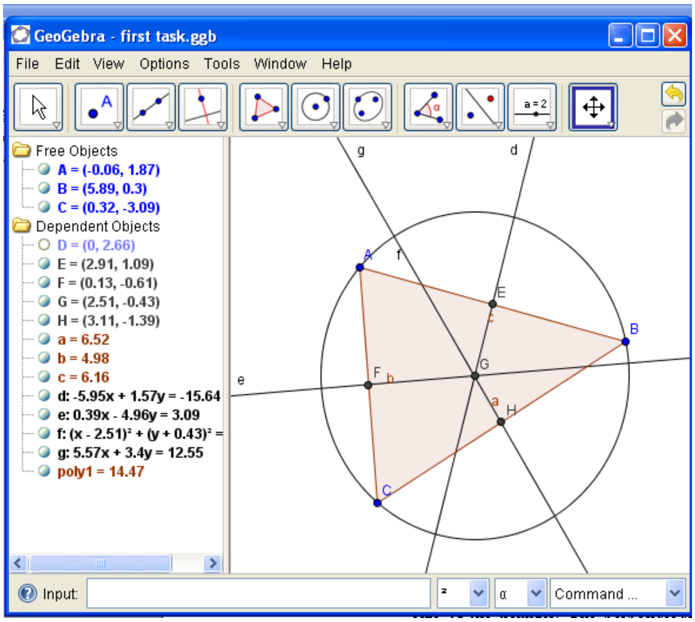
ABSTRACT. *The software GeoGebra is very useful in teaching geometry in middle school or high school classes because it helps students to visualize geometric figures studied and to distinguish between general properties and those resulting from any cases in which a geometric figure was realized. Here are some examples where the using of GeoGebra will be helpful*

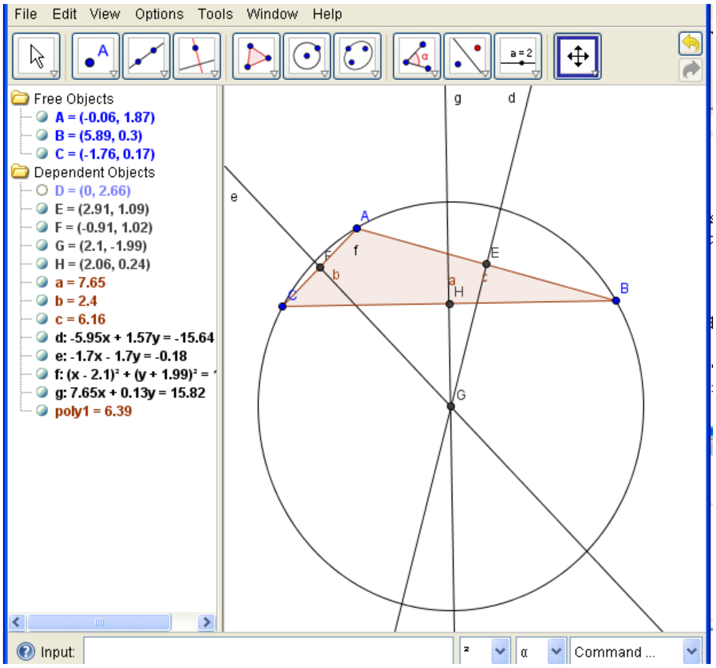
1. The Circumscribed Circle of a Triangle

Teaching the circumscribed circle of a triangle to 6th graders raises some technical problems regarding the accuracy of the drawing in students' notebooks. Due to the imperfection of the tools or to the lack of attention, their circumscribed circle does not contain all the vertices of the triangle, setting the wrong impression that their triangle was not the right one for the circle.

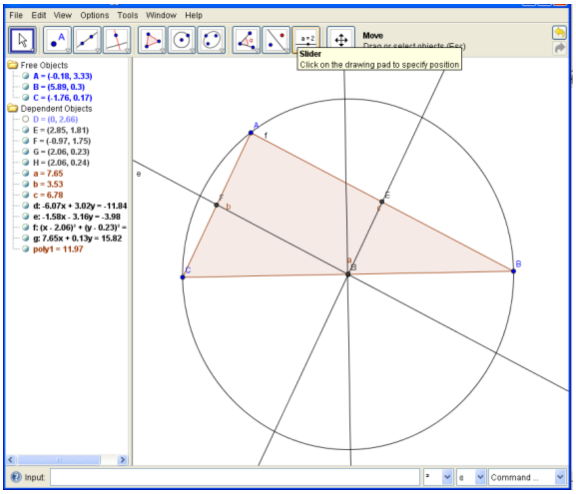
Using GeoGebra when teaching this topic will help students understand that every triangle has a circumscribed circle and its center can be found by intersecting the three perpendicular bisectors of the sides. The steps in doing that will be:

1. Create an arbitrary triangle ABC;
2. Construct the perpendicular bisector for two of its sides and name their intersection point O;
3. Construct the perpendicular bisector of the third side and notice that this one contains point O, regardless of the size or the type of the triangle ABC. For this, click the "move" button, select one of the vertices and drag it on the screen, changing the shape and the size of the triangle. The perpendicular bisectors will change their position with the sides, without changing the fact that O belongs to all three of them;
4. Construct the circle with center O through vertex A and notice that the circle contains also vertices B and C. Perform the "dragging test" to verify that this will happen regardless of the size or shape of the triangle.





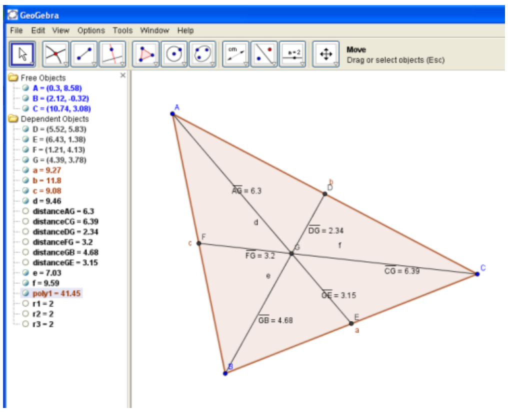
When the drawing is complete, we can study what happens if the triangle is right-angled or obtuse-angled, by dragging a vertex and noticing the position of the circumcenter.



The same steps can be followed for constructing the incenter of the triangle, but replacing perpendicular bisectors of the sides with angle bisectors.

It is known that we teach 6th graders about the center of mass too, but without being able, until 7th grade, to prove that the three medians of the triangle intersect and distance between their intersection point and each vertex is 2/3 of the corresponding median. We can use the measuring tools of GeoGebra to verify this statement, until the students will know enough to prove it. Here are the steps to follow:

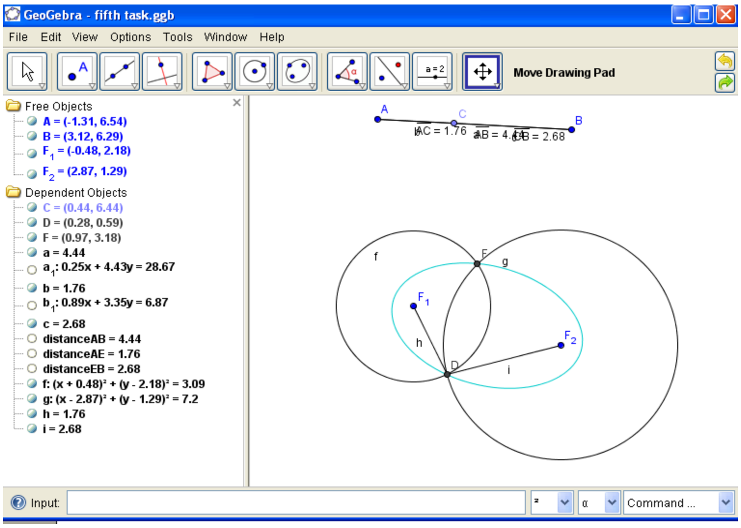
1. Create an arbitrary triangle ABC;
2. Find the midpoints of each side and connect them to the opposite vertex, constructing the medians [BD], [AE], [CF];
3. Mark the intersection point of two of the medians by G and perform "the dragging test" to prove that the third median contains point G;
4. Measure the distances AG, GE, BG, GD, CG, GF and, in the "Input" area, define the ratios AG/GE, BG/GD and CG/GF, noticing that all three of these ratios will always have the value 2, even if we modify the size or the shape of the triangle.



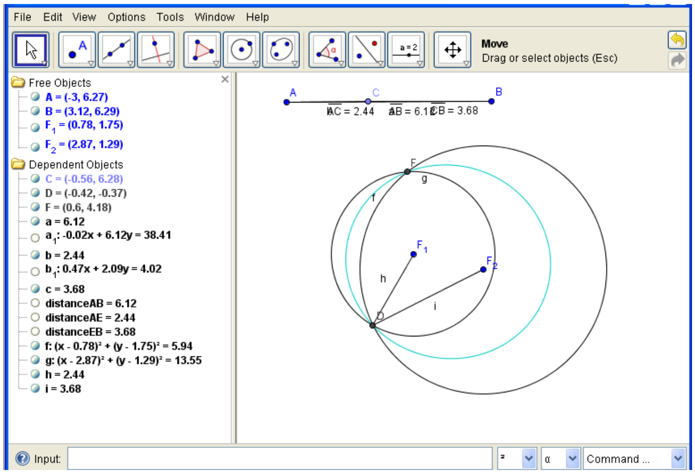
2. The ellipse as a locus

For high school students studying analytic geometry, GeoGebra can be used as a tool to better view the locus. For example, representing the ellipse as the locus of all points P in the plane such that the sum of the distances from P to two fixed points F_1 and F_2 is a given positive constant, can be visualized better on the GeoGebra screen.

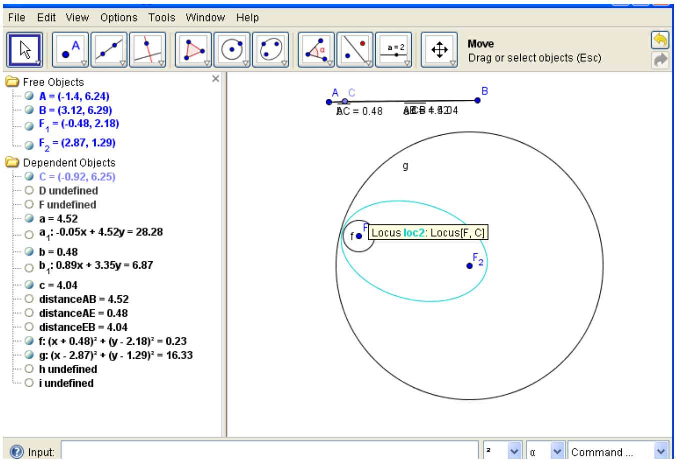
1. Plot the points F_1 and F_2, the foci of the ellipse, and measure the length "a" of the segment F_1F_2.
2. In order to visually represent the constant sum $PF_1 + PF_2$, plot the points A and B such as $F_1F_2 < AB$ and $C \in (AB)$, than measure b = AC and c = CB.
3. Draw the circle with center F_1 and radius b and the circle with center F_2 and radius c. Since $F_1F_2 < AC + BC$, the two circles will intersect in two different points, D and E. This way, $DF_1 = b$ and $DF_2 = c$, so $DF_1 + DF_2 = b + c = AB$, and also $EF_1 + EF_2 = b + c = AB$.
4. Locus of point D in dependence of point C, than locus of point E in dependence of point C will draw the ellipse.
5. Draw segments DF_1 and DF_2 to help students visualize the constant sum of the two lengths.
6. Drag point C along the segment AB. While one of the radius will increase, the other will decrease, but points D and E will move around the ellipse, which will remain unchanged.



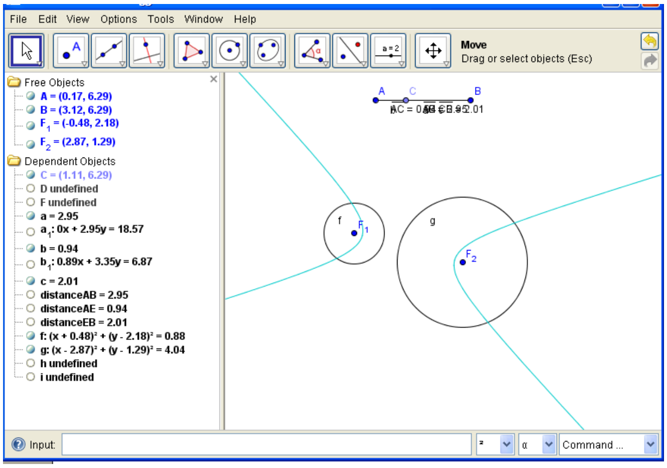
By dragging F_1 or F_2 on the screen, the students will notice that, if the distance between the two foci becomes smaller, the difference between major axis and minor axis will be smaller, too, so the ellipse will be "rounder", and, in exchange, when they increase the distance between the foci, the ellipse will become long and thin.



While dragging point C along the segment AB, the students may notice that, if C is to close to A or B ($|c - b| \geq F_1F_2$), the two circles do not intersect, so the points D and E do not exist.



The students might also notice that, by dragging point A to close to B, the length of AB will become smaller than F_1F_2, so the locus will not be an ellipse anymore, but a hyperbola:



The use of GeoGebra program increases students' interest in mathematics, improves the accuracy of the geometrical figures constructed and significantly decreases the amount of time in which the teacher could illustrate the same concept with different figures.

Content of the work

1 The Circumscribed Circle of a Triangle
2 The ellipse as a locus

Bibliography

Palmira Ronchi - *eTwinning Ideas for Maths , Learning Lab, on http://learninglab.etwinning.net*