

# Separating the Roots of an Equation

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**ABSTRACT** *In the present paper I set out to exemplify the way in which GeoGebra can be used to solve a classic mathematic problem: separating the real roots of a function depending on the values of a real parameter. I will also show how this can be an opportunity to synthesize more knowledge regarding the variation of real functions with real parameters: the limits in the extremities of the domain, monotony etc.*

## 1. The problem to solve

Determine the number of real roots of the equation:

$$mx^3 - 2x^2 - (4m - 1)x + 1 = 0$$

and establish the intervals in which they are found depending on the value of the real parameter  $m$ .

## 2. Preliminaries

Whatever method we choose it is indicated to separate the real parameter  $m$ . So we write the equation in this form:

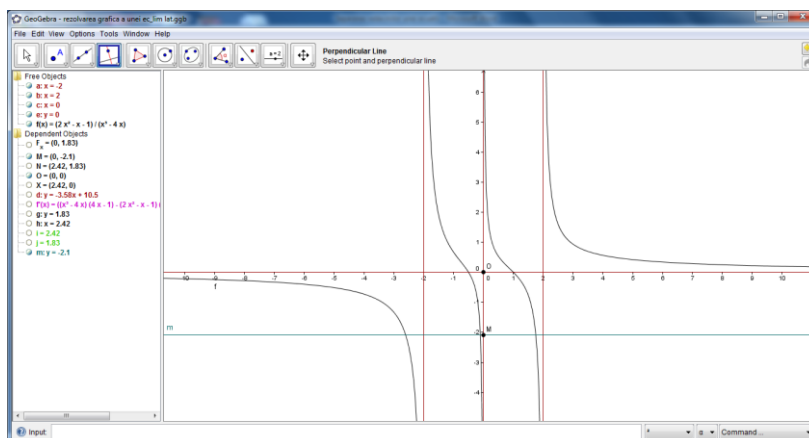
$$m(x^3 - 4x) = 2x^2 - x - 1.$$

We verify through direct calculations that  $x = -2, x = 0, x = 2$  do not verify the equation and therefore we can write it in the equivalent form:

$$\frac{2x^2 - x - 1}{x^3 - 4x} = m \quad (1)$$

### 3. The graphical solution using GeoGebra

Open a GeoGebra file and type in the input bar (Input)  $(2x^2-x-1)/(x^3-4x)$ . Press Enter. You will get the geometric representation of the graph of the function we will name f. Type, one at a time, in the input bar  $x=-2$ ,  $x=0$ ,  $x=2$  and  $y=0$ . Color, for example, in red these line segments (right click each of them, choose Properties, select Color and choose the color). The colored line segments are, as you have surely noticed, the asymptotes to the graph of the function. Now left click on the second menu at the top (that says A), choose New Point and left click a random point of the  $Oy$  axis. The point will be automatically re-named A. Right click this point and choose Rename in the menu that appears. In the window that appears type M, which will be the name of the point. Left click the marked corner of the fourth menu at the top. In the new menu that appears select Parallel Line. Now left click point M and then the  $Ox$  axis. You should get a line segment parallel to the  $Ox$  axis that the program will name a. Rename this line segment m. I suggest you color it turquoise. If everything went well you should have a drawing such as this:



In the algebra window in the left you will not have, for now, all the elements of this drawing. Now left click on the first menu at the top (the one with the arrow) and then left click point M. If you do not release and move the mouse you will notice M moving on the  $Oy$  axis and with it the m line segment. You can now draw the following conclusions:

I) if  $m \in (-\infty, 0)$ , then the equation has three real roots  $x_1, x_2, x_3$  with  $x_1 < -2$ ,  $-2 < x_2 < 0$ ,  $0 < x_3 < 2$

II) if  $m = 0$ , then the equation has two real roots  $x_1 = -\frac{1}{2}, x_2 = 1$  (notice that in this case we get the second degree equation  $2x^2 - x - 1 = 0$ ).

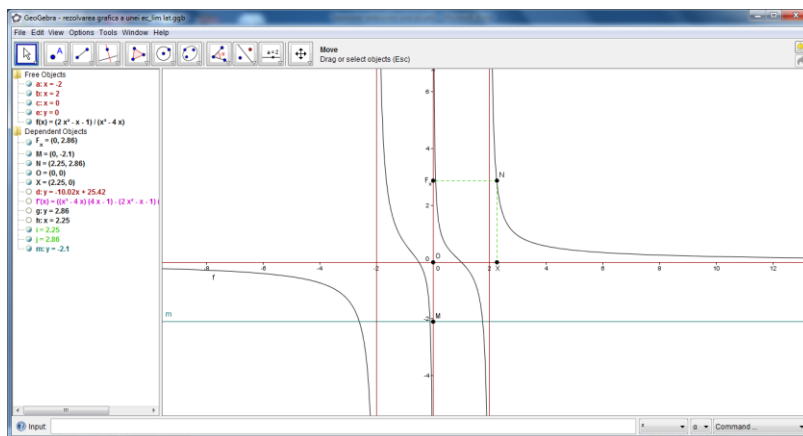
III) if  $m \in (0, \infty)$ , then the equation has three real roots  $x_1, x_2, x_3$  with

$$-2 < x_1 < 0, \quad 0 < x_2 < 2, \quad x_3 > 2$$

#### 4. Other graphical readings

a) The limits of the function at the extremities of the intervals that make up the domain

Choose a new point on the function graph. Rename this point to N and through it, create parallels to the two axes. Left click the right bottom corner of the second top menu. From the menu that appears select Intersect Two Objects. Left click on the parallel to  $Ox$  that you have built through N and then on the  $Ox$  axis. This will create a point. Rename it to X and build the NX line segment (third menu at the top). Hide the parallel through N to  $Ox$  (right click on it and left click on Show Object). You are left only with the NX line segment. Right click it, choose Properties, then Style and choose the dotted line. Now color the NX line segment to green. Follow the same steps to build the intersection between the parallel through N to  $Ox$  with the  $Oy$  axis. Name the point you obtain  $F_x$ . If things went well you should have a drawing that looks something like this:

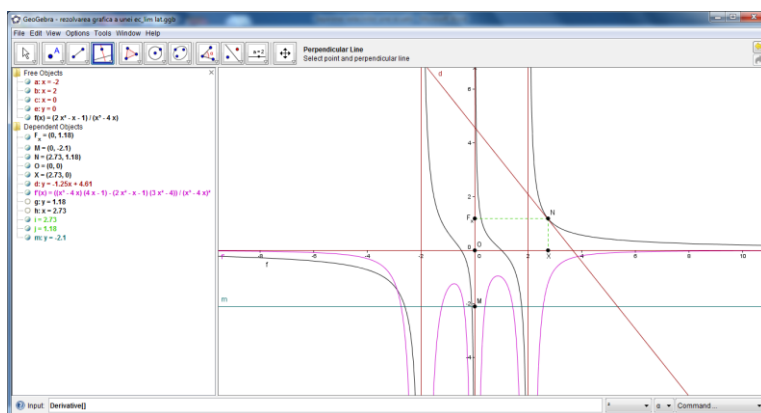


Move point N on the function graph. You will get the following conclusions:

$$\lim_{x \rightarrow -\infty} f(x) = 0, \quad f(-2-0) = -\infty, \quad f(-2+0) = \infty, \quad f(0-0) = -\infty, \quad f(0+0) = \infty, \\ f(2-0) = -\infty, \quad f(2+0) = \infty, \quad \lim_{x \rightarrow \infty} f(x) = 0.$$

## b) The geometric interpretations of the derivative

Left click on the arrow of the Command textbox. From the appearing list choose Derivative. This word will now be in the input bar followed by brackets. Write  $f$  between these brackets and press Enter. GeoGebra has calculated the derivative of the  $f$  function showing it in the algebra window and has drawn the geometric representation of its graph. Color this representation to purple. Now left click on the marked corner of the fourth top menu. From the new menu that appears select Tangents. Left click point  $N$  and then  $f$ . GeoGebra has built the tangent to the graph of  $f$  in point  $N$ . Now color this tangent. You should have a drawing like this one:



Move point  $N$  on the function graph. Notice that the slope of the tangent is negative on each interval of the domain, the derivative is also negative (the geometric representation of its graph is under the  $Ox$  axis) and the function is monotonically decreasing. You can also observe the concavity – convexity intervals.

## 5. Comments

The GeoGebra file created above facilitates graphical readings that are difficult to make otherwise. If you want to have such a lesson I believe it is best to create the file in the classroom. This way you will also be contributing to the development of the students' digital competences. You will also be giving a practical example of Open Source educational software. I also believe that you will be giving the students a new model to study mathematics. Of course, it is indicated that you also study the above equation using Rolle's Theorem. You can study the monotony and convexity – concavity properties by studying the first and second derivative. By making a parallel between the results you obtain you will give your lesson value.