The study of fundamental elements of functions

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ABSTRACT : *This paper presents several applications made for teaching Mathematics in school with the use of "GeoGebra" mathematics software. These applications help the students visualize some properties of the functions or deduce the mathematical conditions under which required properties are present. At the same time, these applications give the opportunity to make similar constructions with different functions or fields*

1 Finding the definition domain of a given function.

This application is appropriate for the introductory chapter of functions.

We consider the function $f:(-1,2)\to R \quad f(x)=2^x$

Moving a mobile point on the graphic of a function, the definition domain will be shown by the trace of the abscissa of the point, marked on the Ox axis.

Animation: In MOVE, mark parameter t and push left or right arrow.

No.	Name	Definition	Algebra
1	Number t		t = -1
2	Point M	(t, 0)	M = (-1, 0)
3	Function f		f(x) = 2^x
4	Number a		a = -1
5	Number b		b = 2
6	Function g	Function f(x) on the interval [a, b]	g(x) = 2^x
7	Point A	(a, f(a))	A = (-1, 0.5)
8	Point B	(b, f(b))	B = (2, 4)
9	Point P	(t, f(t))	P = (-1, 0.5)
10	Segment h	Segment[P, M]	h = 0.5
11	Point Q	(t, 0)	Q = (-1, 0)
12	Point X1	(a, 0)	X1 = (-1, 0)
13	Point X2	(b, 0)	X2 = (2, 0)
14	Text text1		text1 = "D_f=............"

2 The connection between x and f(x).

This application is appropriate for the introductory chapter of functions.

We considered the function $f:(0,\infty)\to R \quad f(x)=\ln x$

The program will show the f(x) value, for every x from definition domain.

Animation: Using the Navigation bar for construction steps, a point on the graphic of the function is built using an x value from the function domain. A vertical interrupted line through the value x crosses the Ox axis, and intersects the curve. Another horizontal interrupted line will pass through that point on the curve, and will intersect the Oy axis, showing f(x) value.

In MOVE, mark parameter d and push left or right arrow.

No.	Name	Definition	Algebra
1	Number d		d = 3
2	Function f		f(x) = ln(x)
3	Point A	(d, 0)	A = (3, 0)
4	Point X	(d, 0)	X = (3, 0)
5	Point B	(d, f(d))	B = (3, 1.1)
6	Segment e	Segment[A, B]	e = 1.1
7	Point C	(0, f(d))	C = (0, 1.1)
8	Segment g	Segment[B, C]	g = 3
9	Point Y	(0, f(d))	Y = (0, 1.1)

3 The connection between x and f(x).

This application is appropriate for the introductory chapter of function.

We considered sample function $f:R\rightarrow R \quad f(x)=x^3+3x^2+2$

We are finding x value, for f(x) given value.

Animation: Using Navigation bar for construction steps, a point on the graphic of function is built using a f(x) value from the codomain. A horizontal interrupted line through the value f(x) crosses the Oy axis, and intersects the curve in one or more points. Another vertical interrupted line will pass through each found point on the curve, and will intersect the Ox axis, finding the appropriate x values.

In MOVE, mark parameter m and push left or right arrow.

No.	Name	Definition	Algebra
1	Function f		f(x) = x³ + 3 x² + 2
2	Number m		m = 4
3	Point A	Point on yAxis	A = (0, -1)
4	Point B	Point on xAxis	B = (-1, 0)
5	Vector u	Vector[(0,0), A]	u = (0, -1)
6	Vector v	Vector[(0,0), B]	v = (-1, 0)
7	Point M	(0, m)	M = (0, 4)
8	Line a	Line through M with direction v	a: y = 4
9	Point C	intersection point of f(x), a	C = (-2.73, 4)
9	Point D	intersection point of f(x), a	D = (-1, 4)
9	Point E	intersection point of f(x), a	E = (0.73, 4)
10	Segment b	Segment[M, C]	b = 2.73
11	Segment k	Segment[M, E]	k = 0.73
12	Segment l	Segment[M, D]	l = 1
13	Line c	Line through C with direction u	c: x = -2.73
14	Line d	Line through D with direction u	d: x = -1
15	Line e	Line through B with direction v	e: y = 0
16	Line i	Line through E with direction u	i: x = 0.73
17	Point X_1	intersection point of c, e	X_1 = (-2.73, 0)
18	Point X_2	intersection point of d, e	X_2 = (-1, 0)
19	Point X_3	intersection point of i, e	X_3 = (0.73, 0)
20	Segment g	Segment[C, X_1]	g = 4
21	Segment h	Segment[D, X_2]	h = 4
22	Segment j	Segment[E, X_3]	j = 4

4 Finding the range of functions restricted to an interval.

This application builds the range of restricted function to an interval .

We start from a function $f : R \to R$, $f(x) = -x^2 + 6x - 8$ and firstly we emphasize the restriction of this function to [2,5](d=2 and e=5) and for each x value from this interval we mark in red the trace of the point corresponding to the f(x) value on Oy axis. In this way , we obtain a segment representing a range of this restricted function on Oy.

Animation: In MOVE, mark parameter s and push left or right arrow.

No.	Name	Definition	Algebra
1	Number s		s = 2.48
2	Number d		d = 2
3	Number e		e = 5
4	Number a		a = -1
5	Number b		b = 6
6	Number c		c = -8
7	Function f	f(x) = a x² + b x + c	f(x) = -x² + 6 x - 8
8	Point C	Point on yAxis	C = (0, 3)
9	Point D	Point on yAxis	D = (0, 0)
10	Point E	(d, 0)	E = (2, 0)
11	Point F	(e, 0)	F = (5, 0)
12	Vector u	Vector[(0,0), C]	u = (0, 3)
13	Line i	Line through E with direction u	i: x = 2
14	Line j	Line through F with direction u	j: x = 5
15	Function g	Function f(x) on the interval [d, e]	g(x) = -x² + 6 x - 8
16	Number m	(f(d) + f(e) - abs(f(d) - f(e))) / 2	m = -3
17	Number n	(f(d) + f(e) + abs(f(d) - f(e))) / 2	n = 0
18	Point S	(s, f(s))	S = (2.48, 0.73)
19	Point Q	(0, f(s))	Q = (0, 0.73)
20	Point T	(s, 0)	T = (2.48, 0)
21	Segment a_1	Segment[S, T]	a_1 = 0.73
22	Segment b_1	Segment[S, Q]	b_1 = 2.48
23	Number t	(-b / 2 a - d) / (e - d)	t = 0.33
24	Number v	If[t > 0 ∧ t < 1, f((1 - t) d + t e), m]	v = 1
25	Number p	Min[v, m]	p = -3
26	Number q	Max[v, n]	q = 1
27	Point A	(0, p)	A = (0, -3)
28	Point B	(0, q)	B = (0, 1)
29	Segment h	Segment[A, B]	h = 4

5 Positive and negative values of a given function.

This application allows covering a curve and at the same time observing the sections of the curve where a function has a positive or a negative value.

We considered a sample function $f : R \to R$, $f(x) = x^3 - 3x^2 + 2$

Animation: In MOVE, mark parameter s and push the left or right arrow. A blue or red trace will appear as the function takes positive or negative values. In MOVE, mark parameter t and push left or right arrow. The blue or red trace will mark the zone of the domain that corresponds to positive or negative function values.

No.	Name	Definition	Algebra
1	Function f		$f(x) = x^3 - 3x^2 + 2$
2	Number s		s = 3.32
3	Number r		r = 5
4	Point R	(r, f(r))	R = (5, 52)
5	Point A		A = (0, 1)
6	Vector u	Vector[(0,0), A]	u = (0, 1)
7	Point B		B = (1, 0)
8	Vector v	Vector[(0,0), B]	v = (1, 0)
9	Line a	Line through A with direction u	a: x = 0
10	Line b	Line through B with direction v	b: y = 0
11	Line g_1	Line through R parallel to a	g_1: x = 5
12	Line f_1	Line through R parallel to b	f_1: y = 52
13	Point E	intersection point of f_1, a	E = (0, 52)
14	Point J	intersection point of g_1, b	J = (5, 0)
15	Segment h_1	Segment[R, J]	h_1 = 52
16	Segment i_1	Segment[R, E]	i_1 = 5
17	Point X_1	intersection point of f(x), b	X_1 = (-0.73, 0)
17	Point X_2	intersection point of f(x), b	X_2 = (1, 0)
17	Point X_3	intersection point of f(x), b	X_3 = (2.73, 0)
18	Number t		t = 3.51
19	Number p	f(t)	p = 8.28
20	Number p_1	f(s)	p_1 = 5.53
21	Number m	If[$p \leq 0$, p]	m undefined
22	Number m_1	If[$p_1 < 0$, p_1]	m_1 undefined
23	Number n	If[$p > 0$, p]	n = 8.28
24	Number n_1	If[$p_1 > 0$, p_1]	n_1 = 5.53

No.	Name	Definition	Algebra
28	Line d	Line through M_1 with direction v	d undefined
29	Point L	intersection point of b, c	L undefined
30	Segment e	Segment[L, M_1]	e undefined
31	Point K	intersection point of d, a	K undefined
32	Segment g	Segment[M_1, K]	g undefined
33	Line b_1	Line through N_1 with direction v	b_1: y = 5.53
34	Line c_1	Line through N_1 with direction u	c_1: x = 3.32
35	Point C	intersection point of b_1, a	C = (0, 5.53)
36	Segment d_1	Segment[N_1, C]	d_1 = 3.32
37	Point D	intersection point of c_1, b	D = (3.32, 0)
38	Segment e_1	Segment[N_1, D]	e_1 = 5.53
39	Point M	(t, m)	M undefined
40	Point N	(t, n)	N = (3.51, 8.28)
41	Line h	Line through M with direction v	h undefined
42	Line i	Line through M with direction u	i undefined
43	Point F	intersection point of i, b	F undefined
44	Segment j	Segment[M, F]	j undefined
45	Point G	intersection point of h, a	G undefined
46	Segment k	Segment[M, G]	k undefined
47	Line l	Line through N with direction v	l: y = 8.28
48	Line q	Line through N with direction u	q: x = 3.51
49	Point H	intersection point of q, b	H = (3.51, 0)
50	Point I	intersection point of l, a	I = (0, 8.28)
51	Segment a_1	Segment[N, I]	a_1 = 3.51
52	Segment j_1	Segment[N, H]	j_1 = 8.28

6 Families of parabolas.

This application allows to visualize the fixed point for a family of parabolas.

Animation: In MOVE, mark parameter m and push left or right arrow.

No.	Name	Definition	Algebra
1	Number m		m = 1.6
2	Function f	$f(x) = x^2 - 2 (m + 1) x + 2 m$	$f(x) = x^2 - 2 (1.6 + 1) x + 2 * 1.6$

7 The family of parabolas that have their peaks on the other parabolas.

For each parabola from a family of parabolas (obtained for a different value of a parameter m(or n)) the trace of the peak is marked in red (or in blue) and describes another parabola (or a line)

Animation: In MOVE, mark parameter m (or n) and push left or right arrow.

No.	Name	Definition	Algebra
1	Number m		m = 2.62
2	Function f	$f(x) = x^2 + (2 m - 2) x + 2 - m$	$f(x) = x^2 + (2 * 2.62 - 2) x + 2 - 2.62$
3	Point V	Extremum of f(x)	V = (-1.62, -3.24)
4	Number n		n = 67.4
5	Function g	$g(x) = n x^2 + 2 (n - 1) x + n - 1$	$g(x) = 67.4 x^2 + 2 (67.4 - 1) x + 67.4 - 1$
6	Point P	Extremum of g(x)	P = (-0.99, 0.99)

8 Restrictions on the peak of parabola so that a function increases on an interval (p, ∞).

This application starts with the parabola associated to a function.

The moving of this curve in the convenient place in accordance with initial requests gives the possibility to analyze the order relation between the abscissa of the peak and the number p

Animation: In MOVE, drag the curve with mouse, and put the parabola in a desired place so that the necessary properties are met.

No.	Name	Definition	Algebra
1	Point B	Point on xAxis	B = (1, 0)
2	Vector u	Vector[(0,0), B]	u = (1, 0)
3	Function f		$f(x) = (x - 3.6)^2 + 0.88$
4	Point V	Extremum of f(x)	V = (3.6, 0.88)
5	Line c	Line through B with direction u	c: y = 0
6	Point A	Point on yAxis	A = (0, 1)
7	Vector v	Vector[(0,0), A]	v = (0, 1)
8	Line a	Line through V with direction v	a: x = 3.6
9	Point X_V	intersection point of a, c	X_V = (3.6, 0)
10	Segment b	Segment[V, X_V]	b = 0.88
11	Number s		s = 3
12	Point p	(s, 0)	p = (3, 0)
13	Point N		N = (10, 0)
14	Ray d	Ray through p, N	d: y = 0
15	Line e	Line through p with direction v	e: x = 3
16	Text text1		text1 = "conditii:-b/2a.......p a......0"