# Applications in vectorial geometry and mathematical analysis

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**ABSTRACT.** *This material concernes the graphical representation of different situations which help beginer students in mathematics to better understand certain mathematical chapters. There are seven applications, with different themes about: vectors, continuity of functions, Riemann integration, areas of plain surfaces, and the tangent to a curve. These methodical themes can be aproached in different ways, but we are not concerned about these for the moment. Our interest is focussed on graphical aspects which the GeoGebra software can offer, and we give the construction protocol of each figure and some images of the simulations.*

## 1. An application for vectors

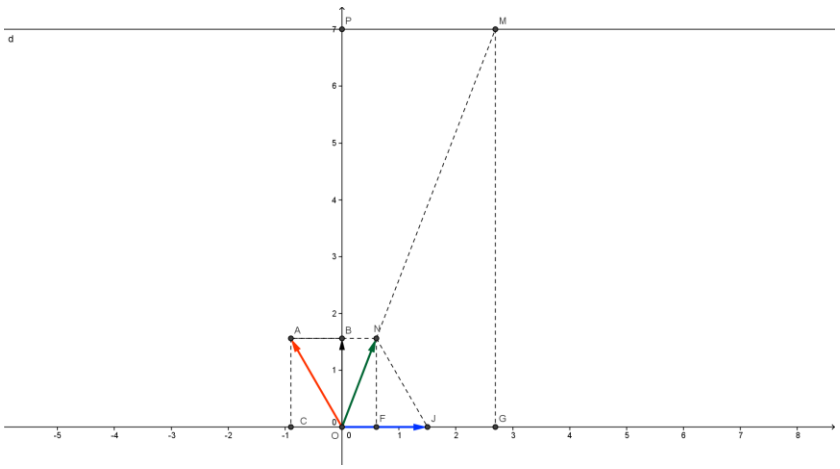
The modelling of the next situation suggests the next simulation**.**

Two swimmers cross simultaneusly, a river, with the width of *p* meters. Both of them swim with the same velocity of *m* m/s.

The velocity of the river is v m/s flowing towards east. One of the swimmers crosses the river making an angle of *a* radians with the direction of flow of the river. The other swimmer crosses the river making an angle of *b* radians with the direction of flow of the river.

Find the time for crossing the river for each swimer

Animation: In Move, mark parameter n and push the Right arrow. Parameter n means ungle AOB, measured in radians.



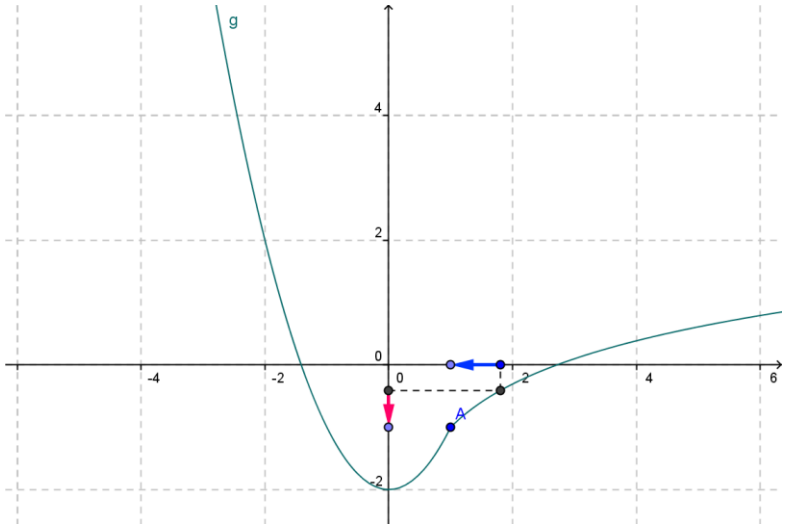

| No. | Name | Definition | Algebra |
|---|---|---|---|
| 1 | Number m | | m = 1.8 |
| 2 | Number n | | n = 1.5 |
| 3 | Number a | | a = -0.52 |
| 4 | Point A | (m sin(a), m cos(a)) | A = (-0.9, 1.56) |
| 5 | Point B | (0, m cos(a)) | B = (0, 1.56) |
| 6 | Point C | (m sin(a), 0) | C = (-0.9, 0) |
| 7 | Segment b | Segment[B, A] | b = 0.9 |
| 8 | Segment c | Segment[A, C] | c = 1.56 |
| 9 | Point O | intersection point of xAxis, yAxis | O = (0, 0) |
| 10 | Vector u | Vector[O, A] | u = (-0.9, 1.56) |
| 11 | Vector w | Vector[O, B] | w = (0, 1.56) |
| 12 | Point N | (m sin(a) + n, m cos(a)) | N = (0.6, 1.56) |
| 13 | Segment e | Segment[A, N] | e = 1.5 |
| 14 | Vector z | Vector[O, N] | z = (0.6, 1.56) |
| 15 | Number p | | p = 7 |
| 16 | Point M | ((m sin(a) + n) p / (m cos(a)), p) | M = (2.69, 7) |
| 17 | Segment g | Segment[N, M] | g = 5.83 |
| 18 | Line d | y = p | d: y = 7 |
| 19 | Point F | (m sin(a) + n, 0) | F = (0.6, 0) |
| 20 | Point G | ((m sin(a) + n) p / (m cos(a)), 0) | G = (2.69, 0) |
| 21 | Segment h | Segment[N, F] | h = 1.56 |
| 22 | Segment i | Segment[M, G] | i = 7 |
| 23 | Point J | (n, 0) | J = (1.5, 0) |
| 24 | Vector v | Vector[O, J] | v = (1.5, 0) |
| 25 | Point P | (0, p) | P = (0, 7) |
| 26 | Segment f | Segment[N, J] | f = 1.8 |

## 2. The continuity in $x_0$ =1, of the function:

$$f: R \to R,\ f(x) = \begin{cases} x^2 - 2, & x < 1 \\ -1, & x = 1 \\ \ln x - 1, & x > 1 \end{cases}$$

Animation: Move, mark parameter k and push the right or left arrow.

| No. | Name | Definition | Algebra |
|---|---|---|---|
| 1 | Function f | If[x ≤ 1, x² - 2] | f(x) = If[x ≤ 1, x² - 2] |
| 2 | Function g | If[x > 1, -1 + ln(x), f(x)] | g(x) = If[x > 1, -1 + ln(x), f(x)] |
| 3 | Point A | | A = (1, -1) |
| 4 | Number k | | k = 1.8 |
| 5 | Point B | (k, 0) | B = (1.8, 0) |
| 6 | Point C | (k, g(k)) | C = (1.8, -0.41) |
| 7 | Point D | (0, g(k)) | D = (0, -0.41) |
| 8 | Point E | Point on yAxis | E = (0, -1) |
| 9 | Vector u | Vector[D, E] | u = (0, -0.59) |
| 10 | Point F | Point on xAxis | F = (1, 0) |
| 11 | Segment a | Segment [C, B] | a = 0.41 |
| 12 | Segment b | Segment [D, C] | b = 1.8 |
| 13 | Vector v | Vector[B, F] | v = (-0.8, 0) |

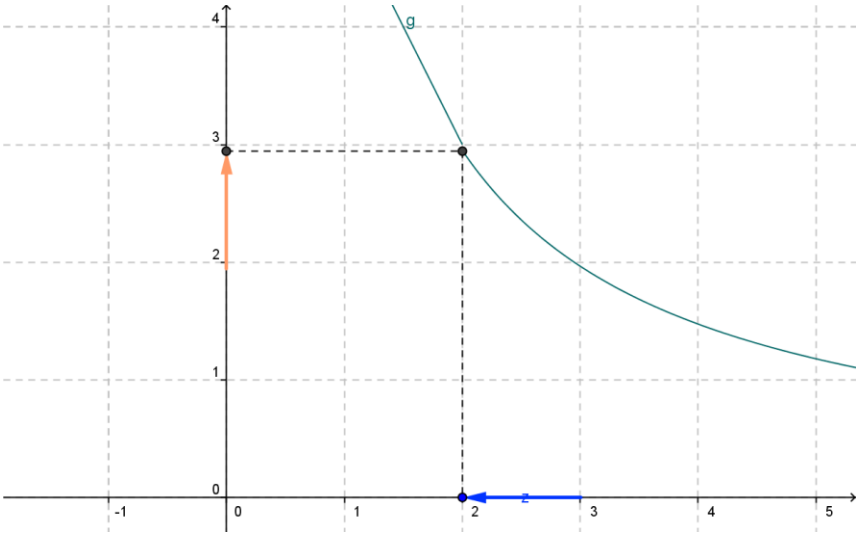



## 3. Left continuity of a function

The function $f : R \to R$, $f(x) = \begin{cases} -2x + 7, & x < 2 \\ 3, & x = 2 \\ \dfrac{59}{10x}, & x > 2 \end{cases}$ is continuous to the left in $\mathbf{x_0} = 2$

but is not continuous in 2.

Animation: In Move, mark parameter k and push the Right or left arrow.

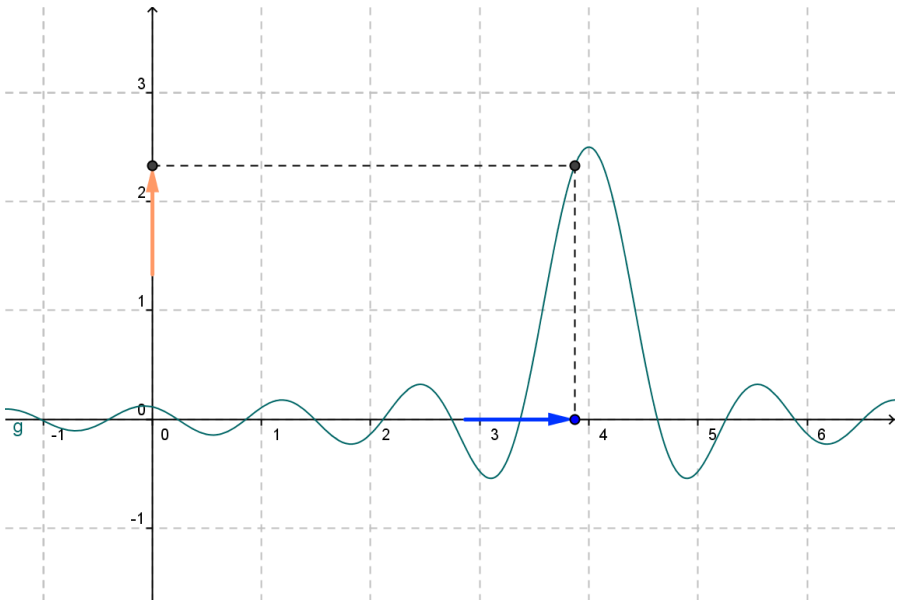



| No. | Name | Definition | Algebra |
|---|---|---|---|
| 1 | Function f | If[x < 2, -2 x + 7] | f(x) = If[x < 2, -2 x + 7] |
| 2 | Function g | If[x > 2, 59 / (10 x), f(x)] | g(x) = If[x > 2, 59 / (10 x), f(x)] |
| 3 | Number k | | k = 2 |
| 4 | Point B | (k, 0) | B = (2, 0) |
| 5 | Point C | (k, g(k)) | C = (2, 2.95) |
| 6 | Point D | (0, g(k)) | D = (0, 2.95) |
| 7 | Segment a | Segment [C, B] | a = 2.95 |
| 8 | Segment b | Segment [D, C] | b = 2 |
| 9 | Point F | (k - 1, 0) | F = (1, 0) |
| 10 | Point G | (0, g(k) - 1) | G = (0, 1.95) |
| 11 | Vector v | Vector[F, B] | v = (1, 0) |
| 12 | Point H | (0, g(k) + 1) | H = (0, 3.95) |
| 13 | Vector u | Vector[H, D] | u = (0, -1) |
| 14 | Vector w | Vector[G, D] | w = (0, 1) |
| 15 | Point I | (k + 1, 0) | I = (3, 0) |
| 16 | Vector z | Vector[I, B] | z = (-1, 0) |
| 17 | Point A | | A = (2, 3) |

## 4 . Discontinuity of a function

$f : R \to R, \quad f(x) = \begin{cases} \dfrac{\sin 5(x-4)}{2x-8}, & x \neq 4 \\ 2, & x = 4 \end{cases}$ is not continuos in point 4.

Animation: Move, mark parameter k and push the right or left arrow.

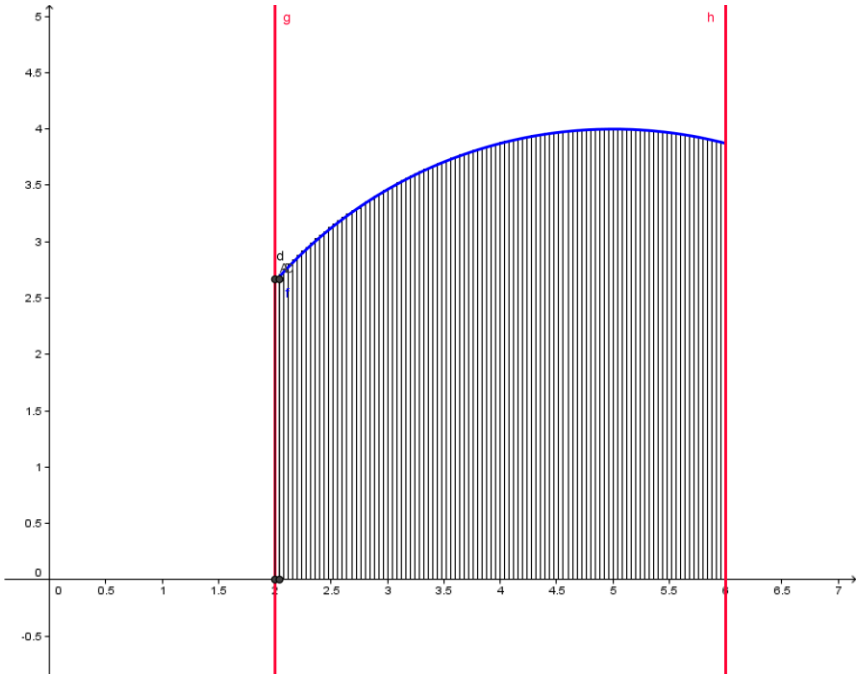



| No. | Name | Definition | Algebra |
|---|---|---|---|
| 1 | Function f | If[x < 4, sin(5 (x - 4)) / (2 x - 8)] | f(x) = If[x < 4, sin(5 (x - 4)) / (2 x - 8)] |
| 2 | Function g | If[x > 4, sin(5 (x - 4)) / (2 x - 8), f(x)] | g(x) = If[x > 4, sin(5 (x - 4)) / (2 x - 8), f(x)] |
| 3 | Number k | | k = 3.87 |
| 4 | Point B | (k, 0) | B = (3.87, 0) |
| 5 | Point C | (k, g(k)) | C = (3.87, 2.33) |
| 6 | Point D | (0, g(k)) | D = (0, 2.33) |
| 7 | Segment a | Segment [C, B] | a = 2.33 |
| 8 | Segment b | Segment [D, C] | b = 3.87 |
| 9 | Point F | (k - 1, 0) | F = (2.87, 0) |
| 10 | Point G | (0, g(k) - 1) | G = (0, 1.33) |
| 11 | Vector v | Vector[F, B] | v = (1, 0) |
| 12 | Point H | (0, g(k) - 1) | H = (0, 1.33) |
| 13 | Vector u | Vector[H, D] | u = (0, 1) |
| 14 | Vector w | Vector[G, D] | w = (0, 1) |
| 15 | Point I | (k + 1, 0) | I = (4.87, 0) |
| 16 | Vector z | Vector[I, B] | z = (-1, 0) |
| 17 | Point A | | A = (4, 1) |

## 5. Riemann integral

Simulation of covering the bounded surface with rectangles, for introducing Riemann sums, and Riemann integrals.

Animation: Move, mark parameter k and push the right or left arrow. Also, if we increase the value of parameter n we will see that the cover of rectangles aproximates entire the surface.



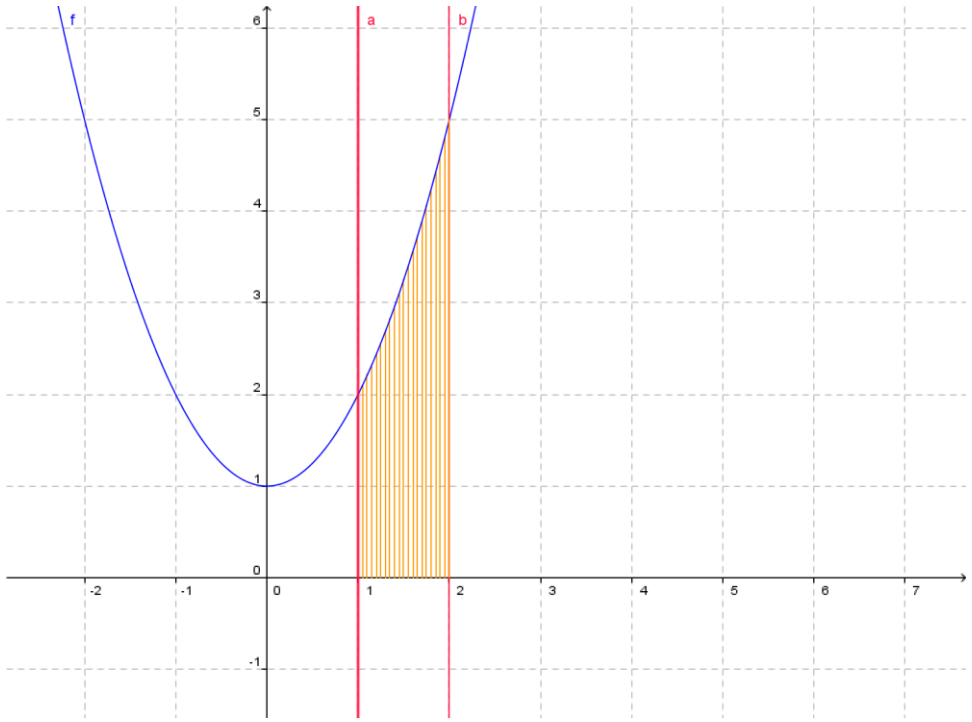

| No. | Name | Definition | Algebra |
|---|---|---|---|
| 1 | Function f | Function sqrt(16 - (x - 5)²) on interval [2, 6] | f(x) = sqrt(16 - (x - 5)²) |
| 2 | Number a | | a = 2 |
| 3 | Number b | | b = 6 |
| 4 | Number n | | n = 100 |
| 5 | Number k | | k = 0 |
| 6 | Point B | (2 + k (b - a) / n, 0) | B = (2, 0) |
| 7 | Point A | (2 + k (b - a) / n, f(2 + (2 k + 1) (b - a) / (2 n))) | A = (2, 2.67) |
| 8 | Segment c | Segment [A, B] | c = 2.67 |
| 9 | Point C | (2 + (k + 1) (b - a) / n, 0) | C = (2.04, 0) |
| 10 | Point D | (2 + (k + 1) (b - a) / n, f(2 + (2 k + 1) (b - a) / (2 n))) | D = (2.04, 2.67) |
| 11 | Segment d | Segment [D, A] | d = 0.04 |
| 12 | Segment e | Segment [D, C] | e = 2.67 |
| 13 | Line g | | g: x = 2 |
| 14 | Line h | | h: x = 6 |

## 6. Areas of plain surfaces

Simulation for calculus of areas of the bounded surface . 6.1, 6.2, 6.3

Animation: Move, mark parameter k and push the right or left arrow.

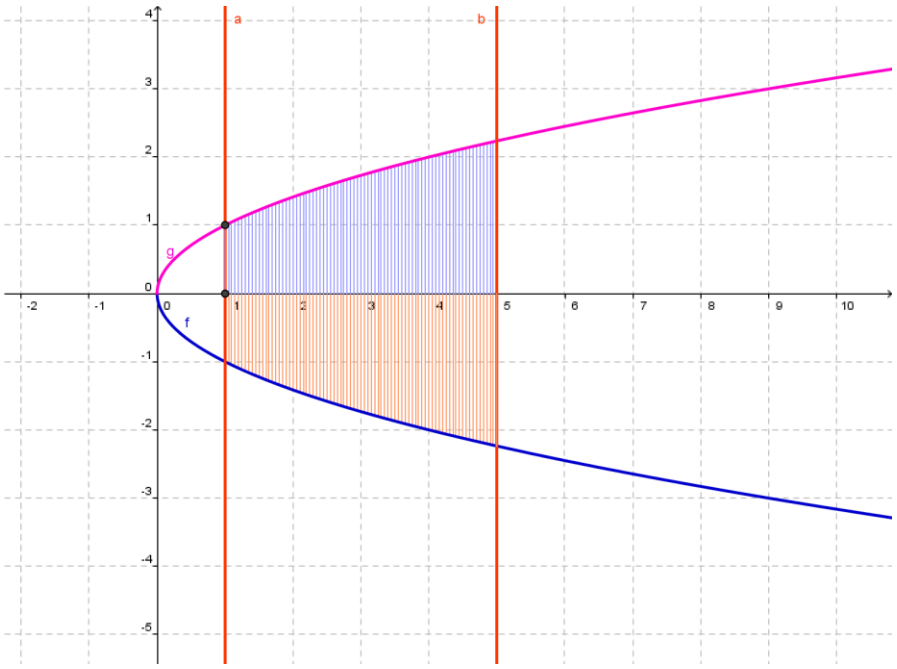
6.1




## 6.1

| No. | Name | Definition | Algebra |
|---|---|---|---|
| 1 | Function f | | $f(x) = x^2 + 1$ |
| 2 | Line a | | a: x = 1 |
| 3 | Line b | | b: x = 2 |
| 4 | Number k | | k = 1 |
| 5 | Point A | (k, f(k)) | A = (1, 2) |
| 6 | Point B | (k, 0) | B = (1, 0) |
| 7 | Segment c | Segment [A, B] | c = 2 |

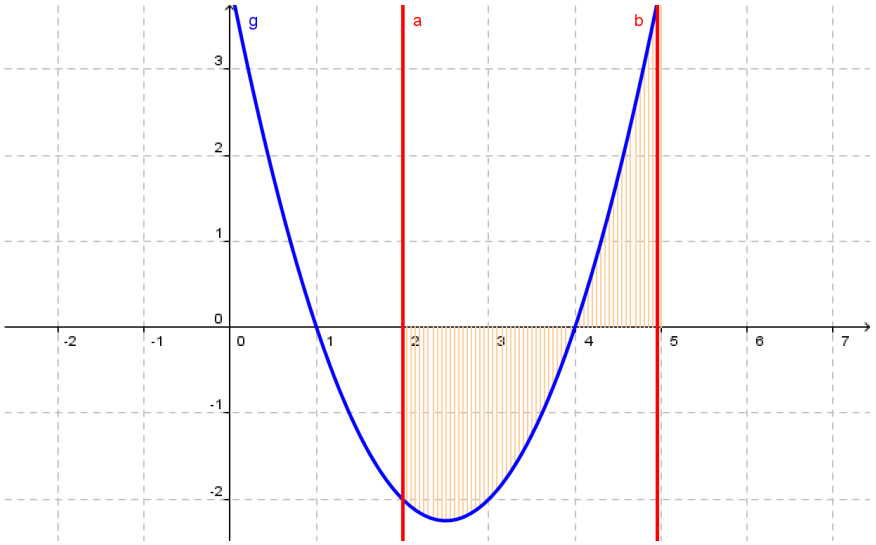
6.2




## 6.2

| No. | Name | Definition | Algebra |
|---|---|---|---|
| 1 | Function f | | f(x) = -sqrt(x) |
| 2 | Line a | | a: x = 1 |
| 3 | Line b | | b: x = 5 |
| 4 | Number k | | k = 1 |
| 5 | Point A | (k, f(k)) | A = (1, -1) |
| 6 | Point B | (k, 0) | B = (1, 0) |
| 7 | Segment c | Segment [B, A] | c = 1 |
| 8 | Function g | | g(x) = sqrt(x) |
| 9 | Number s | | s = 5 |
| 10 | Point C | (s, g(s)) | C = (5, 2.24) |
| 11 | Point D | (s, 0) | D = (5, 0) |
| 12 | Segment d | Segment [C, D] | d = 2.24 |

6.3



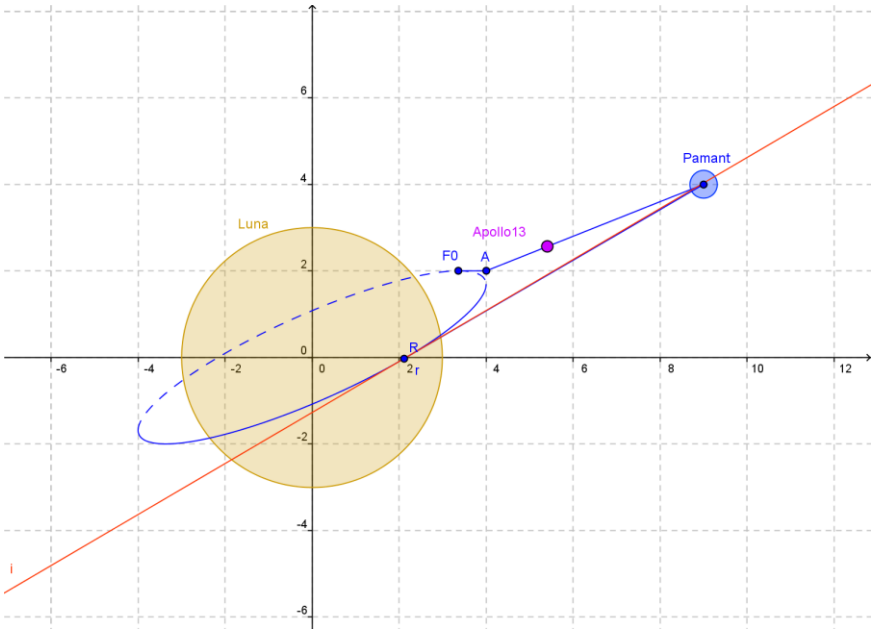

**6.3**

| No. | Name | Definition | Algebra |
|---|---|---|---|
| 1 | Function g | | g(x) = x² - 5 x + 4 |
| 2 | Line a | | a: x = 2 |
| 3 | Line b | | b: x = 4.96 |
| 4 | Number k | | k = 2 |
| 5 | Point A | (k, g(k)) | A = (2, -2) |
| 6 | Point B | (k, 0) | B = (2, 0) |
| 7 | Segment c | Segment [B, A] | c = 2 |

## 7. Application: tangent to a curve

Simulation of hypothetical trajectory around the moon, inspired by the rescuing operation of Apollo 13.

Animation: Move, mark parameter s and push the Right or left arrow.


Pamant
Luna
Apollo13
F0
A
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r
i