Constructions with the ruler and the compass tangential to a circle

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ABSTRACT. *In this paper we presented a problem of construction of a line tangent from a point outside a circle in that circle. Construction difficulty, at first sight, it is time that the circle center is inaccessible. Presentation of the construction is done so with GeoGebra application, benefiting from its facilities, and a solution based on elementary mathematical solutions with ruler and compass construction*

1. The GeoGebra task

Create GeoGebra a file through which to learn how to trace tangents from a given point P at a given circle whose center is inaccessible.

2. The solution to the GeoGebra task

Each construction that we do with the help of the GeoGebra software will be renamed. Notations below are subsequent to renaming.

Consider a given circle C1, and because we do not know the center, we will not make it visible, then an external point to P. From the point P we trace a half line PD, that is secant to the circle C1, point D will not appear in the drawing because it is of no interest.

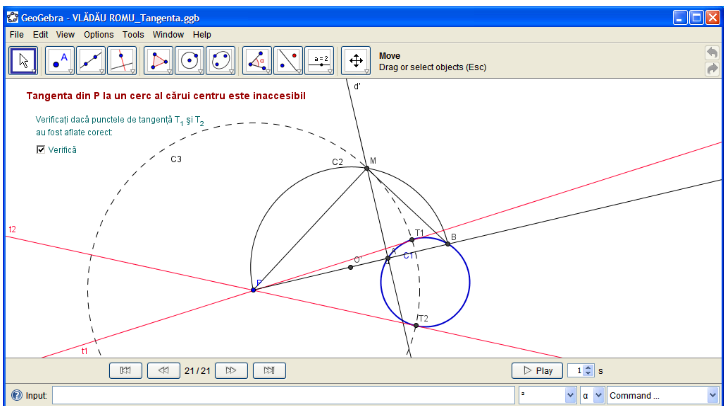


Figure 1. The GeoGebra application

We will find the intersection points of the secant with the circle C1, renaming them, again A and B (we consider A between P and B). We find the middle of the segment PB with the help of the application, calling it O '. This point is not necessary in the following construction, but we highlight it for the scientific arguments of the construction.

We trace the semi-circle starting from P and B (obviously the center O '), denoted by C2, then we trace the right d' perpendicular to the secant PD in point A. We find the point of intersection between it and the diameter C2 further denoted with M.

We draw the circle with the center in P and radius PM, denoted by C3, then we find its points of intersection with the circle C1, denoted T1, T2- respectively.

Certainly, these points are the points sought, so easily verifiable by the application if the tangents are drawn from P to the circle C1.

Finally, the geometric construction is obtained as shown below:

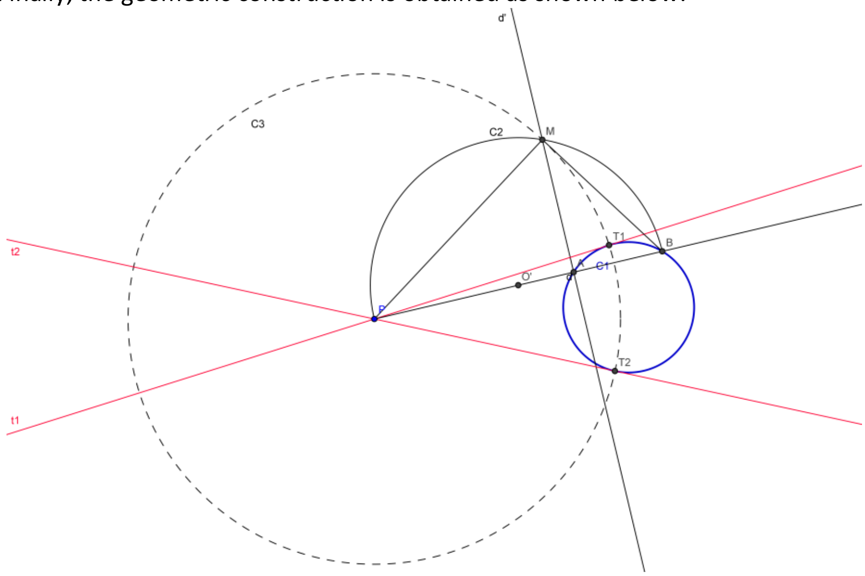


Fig. 1. The starting data were drawn in blue, aiding constructions in black, and the tangents (the result of construction) in red.

3. The mathematical solution

C1 be the circle whose center we do not know (it is inaccessible) and a point outside it, P. A secant taken from P cuts the circle C1 in points A and B.

We can assume that point A is between points P and B without distorting the demonstration.

Be O” the middle of the segment PB and we build the circle with center O' and radius OP ', which we denote by C2. From point A we draw a perpendicular on PB, which will intersect the circle C2 in M. The PMB triangle, inscribed in the circle C2 with PB diameter is rectangular in M, and MA is the height in this triangle.

According to the catheti theorem PMB we have: $PM^2 = PA \cdot PB$ (1).

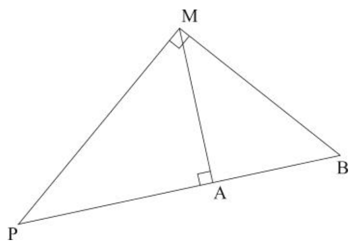
We note with $\rho(P)$ the power of the point P to the circle C2. We have: $\rho(P) = PA \cdot PB = PT_1^2$ (2), where T is the point of contact sought.

From relations (1) and (2) results that $PM^2 = PT_1^2$, hence. PM= PT.

As such, the point of contact sought, T1 is obtained tracing the circle with the center P and radius PM, denoted by C3, which will intersect the circle C1 in point T1 (T2, respectively).

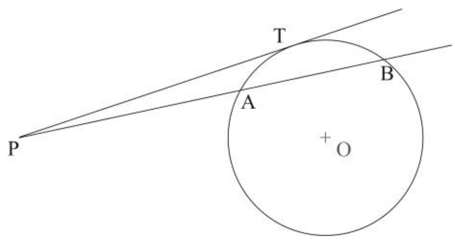
4. Theoretical notions

Construction solution is based on theoretical concepts, namely:
Catheti theorem: In any right triangle, a catheti is average proportional between the hypotenuse and its projection on the hypotenuse.



$$PM^2 = PB \cdot PA$$

The power of a point from a circle:



$$\rho(P) = PA \cdot PB = PT^2$$

5. Comments

GeoGebra file created allows solving problems easily. In solving the problem, although a centre was necessary for the initial circle (the given one), the need is given only by the construction of the circle itself, having no relevance in solving the problem itself.

The secant taken from the point P to the given circle C1 has no constraints, it can be taken anywhere, provided C1 circle is intersected in two points.

The program allows the step by step drawing of the construction, so being of a real help in exposing and understanding it.

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