Some aspects of coupler curves study in the planar four-bar linkage using mathematical-graphical environment GeoGebra

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ABSTRACT. The paper's goal is to highlight the great capability to study the planar linkages using the graphical-mathematical environment GeoGebra. After a briefly presentation of some basic theoretical aspects concerning the four-bar planar linkage, is presented a GeoGebra application for study of the coupler-curves in the planar four-bar linkage. In this context are given information on application's functionality and some its educational valences. At the end of the article are listed some remarks of the author.

1 Preliminary

Perhaps geometric interpretation is one of the most powerful mathematical tools of study in all fields of science in which mathematical model is suitable to accept an such interpretation or in which even solving of the mathematical model allow to be applied a graphic-analytical calculus method.

In recent decades, graphic statics, branch of mechanics, was almost abandoned because its widespread use of the analytical calculus methods, due to upward evolution of the numerical computing power of the computers. In this context, the main disadvantage of the graphical calculus methods is related to the accuracy of the results which is dependent on the precision and accuracy of graphical representations on paper and of the measurements accuracy on them. Thus its came the idea that graphical calculus methods provided by graphic statics are outdated.

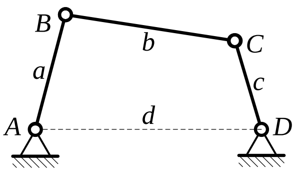
However, it is important to be noted that graphical calculus methods have a major advantage over analytical calculus methods; they allow a full coverage of the problem in a single graphical representation and they make the results to be suggestive and easily interpretable. Also, it is easy to find possible errors that were made during of the problem solving.

The analytical calculus methods do not have these properties; they allow solving the problem in a sequential manner in which each intermediate result is a consequence of the previous result. Although the accuracy of numerical results obtained in purely analytic calculus can be any high, however, in most cases this precision it is useless, because it is insignificant compared with the numerical accuracy of the data technical entry.

The development of the geometric software packages may lead to a revival of the graphical methods, especially because this approach eliminates the main disadvantage which has been above presented. In this context, GeoGebra which is a dynamic and interactive geometry environment, having many powerful instruments, is an excellent example.

2 Theoretical basics on the four-bar planar linkage

In the range of planar mechanisms, the simplest group of mechanisms is four-bar linkages. A four-bar linkage is the simplest movable linkage. It consists of four rigid bodies (which are commonly called *links*), each attached to two others by single joints to form a closed loop as shown in Figure 1.



- A, B, C, D – single joints
- d – length of the fixed link
- a, c – length of the side links
- b – length of the coupler link

Figure 1 - Composition of the planar four-bar linkage

Two of the four joints are fixed, and the connection between them is called *the ground link*, *the fixed link* or, simply, *the frame*. The two links connected with the fixed joints are called *side links*. In terms of mechanical action, one of the side links is the *input link*, i.e., the link to which an external torque is applied to rotate it. The second side link is called the *follower link*, since its motion is completely determined by the motion of the input link.

In this area, if we refer to links lengths, are used the following notations: s - length of shortest link, l - length of longest link, p,q - length of intermediate links. With the previous notations and considering Figure 1, it is possible to write:

$$l = \max\{a,b,c,d\},\ s = \min\{a,b,c,d\},\ p,q \in \{a,b,c,d\} - \{l,s\} \qquad (1)$$

Depending on the nature of the movements they perform side links, these have specific names:

- *Crank* - a side link which revolves relative to the fixed link.
- *Rocker* - a side link which does not revolve to the fixed link.

Therefore, it follows that any four-bar linkage can be assigned to one of below types:

- *Crank-rocker mechanism* - the shorter side link revolves and the other one rocks.
- *Double-crank mechanism* - both of the side links revolve.
- *Double-rocker mechanism* - both of the side links rock.

The input link for crank-rocker mechanism is the crank and for double-crank mechanism and double-rocker mechanism the input link is any side link.

The type of four-bar linkage is found on Grashof's theorem basis. Without we propose a rigorous demonstration of this theorem, below are presented the results of applying this theorem to the classification of the four-bar linkages. There are two cases, depending on the relationship between $s+l$ and $p+q$: for $s+l \le p+q$ (see on Figure 2) and for $s+l > p+q$ (see on Figure 3).

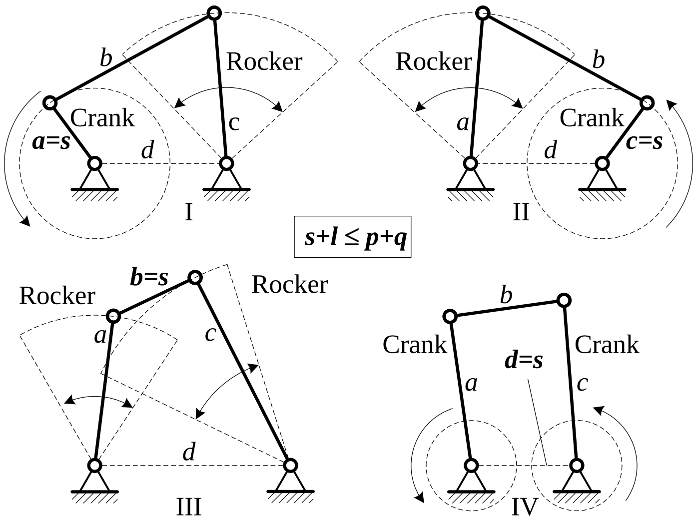


Figure 2 – Types of the four-bar linkage for $s+l \le p+q$

As seen in Figure 2, for $s+l \le p+q$, the type of the four-bar linkage depends on the position of the shortest link: if the shortest link is a side link (Figure 2 I, II) the four-bar linkage is a crank-rocker mechanism, if the shortest link is the coupler link (Figure 2 III) the four-bar linkage is a double-rocker mechanism and if the shortest link is the fixed link (Figure 2 IV), the four-bar linkage is a double-crank mechanism.

For $s+l > p+q$, the type of the four-bar linkage is independent of the position of the shortest link. In this case it is possible to obtain only double-rocker mechanisms.

All the above about the type of the four-bar linkage can be summarized in the following table:

Table 1 – Four-bar linkage types

Type	**Dimensional characterization**	
	$s+l$ vs. $p+q$	**Shortest link position**
Double-crank	$s+l \le p+q$	$s=d$
Crank-rocker	$s+l \le p+q$	$s=a$ or $s=c$
Double-rocker	$s+l \le p+q$	$s=b$
	$s+l > p+q$	Any position

The curve which is generated by a fixed point on coupler link when the input link is moving is called *coupler curve* (see on Figure 3). Thus defined, coupler curve is nothing but the locus of a fixed point on the coupler when point *B* is moving on an arc centered on *A*.

The coupler curves are very important in synthesis of the linkage mechanisms because they are often imposed to obtain a corresponding mechanism.

Roberts–Chebyschev theorem is a very important theorem in designing mechanisms which must be able to reproduce a certain coupler curve. This theorem states *for a given coupler-curve there exist three four-bar linkages, three geared five-bar linkages, and more six-bar linkages which will generate the same path*.

3 GeoGebra application

In order to visualize and to study the coupler curves generated by any four-bar linkage has been created a GeoGebra application. A portion of the application window is presented in Figure 3.

Dimensional characterization of the mechanism can be performed by sliders (called **a**, **b**, **c** and **d** into application). To the right of these sliders are displayed type of the mechanism for a certain combination of the numerical values for a, b, c and d, their description and the mathematical relationships between them.

The user can choose to be displayed simultaneously up to seven coupler curves described by next points:

- M (midpoint of the segment BC);
- E and F on the line (BC) and symmetrical with respect to M;
- I and J on the perpendicular to the line (BC) in E and symmetrical with respect to E;
- G and H on the perpendicular to the line (BC) in F and symmetrical with respect to F.

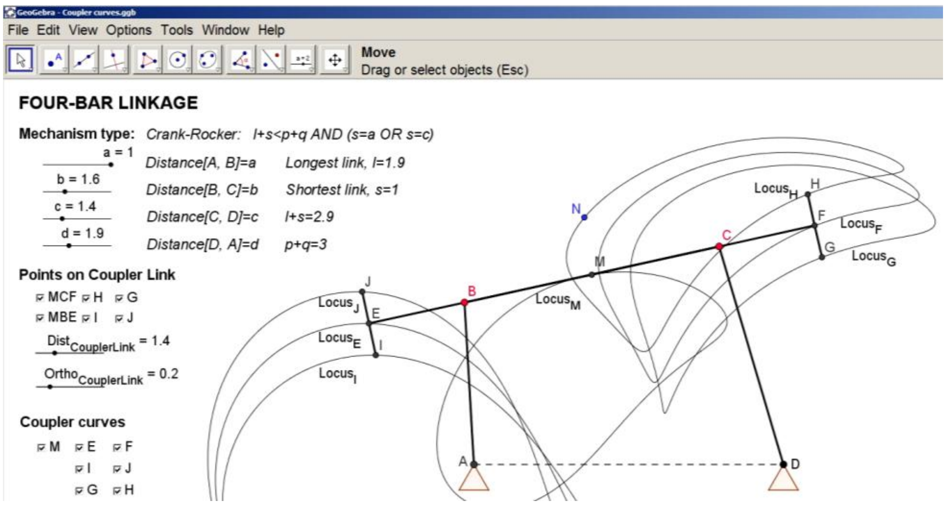


Figure 3 – GeoGebra application

The position of the points E and F is controlled by [illegible] slider and the positions of the points I, J, G and H are controlled by [illegible] slider. Selection of one or more curves (and corresponding points) is made by selecting of the appropriate Check Boxes (i.e. in case from Figure 4 were selected all Check Boxes).

The user can self-control the level of knowledge by predicting the type of mechanism for a certain combination of lengths of links using data summarized in Table 1, then performing a check of the prediction using GeoGebra application.

Perhaps more important than a self-checking of the knowledge is the capability to allow user to find by successive attempts a mechanism able to generate a coupler curve with prescribed shape. Also, it is possible to find interpolation curve equation for a specified portion from coupler curve. In this context, in Figure 4 is presented the Hoekens linkage ($a=1, b=2.5$, $c=2.5, d=2, CF=2.5$) that converts rotational motion to approximate straight-line motion.

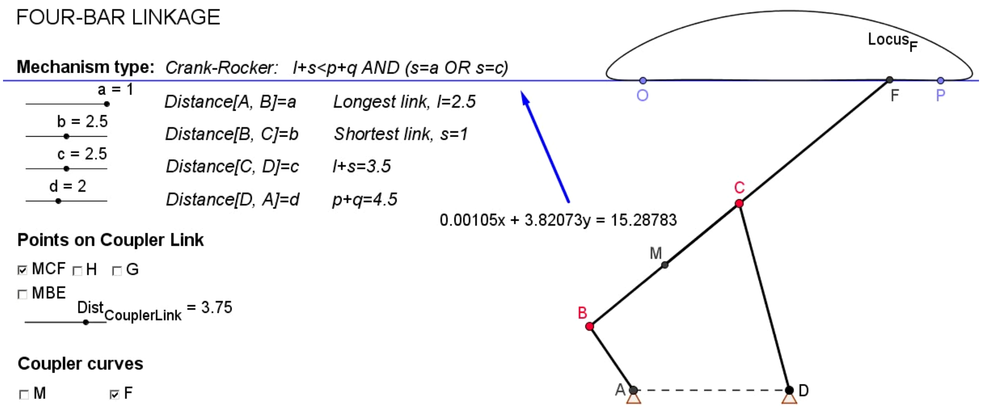


Figure 4 – Hoekens linkage

As seen in Figure 4, the Hoekens mechanism generates a coupler curve with a portion very close to a straight line parallel with segment AD.

4 Remarks

First is to note that although there are numerous programs for the analysis of mechanisms, the GeoGebra environment provides all the tools needed to building of advanced applications in this area.

In this context, GeoGebra environment can become not only a dynamic environment for learning and teaching but also an environment where it is possible to build advanced technical studies in areas such as synthesis and analysis mechanisms, mechanics, strength structures statics, etc.

The above presented GeoGebra application is a part of a larger development whose purpose is the design and analysis of the planar linkage mechanisms.

5 References

[1] **Norton, R.** – *An introduction to the synthesis and analysis of mechanisms and machines*, McGraw Hill Series in Mechanical Engineering, 1999

[2] **Hohenwarter, J., Hohenwarter, M.** - *Introduction to GeoGebra*, 2008