New methods of teaching and learning mathematics involved by GeoGebra

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Abstract: *Archimedes drew his figures on beach sand, mud or ash on a floor or put on his body, previously anointed with oil; on his body the figures were drawn with nails. When the Roman general Marcellus conquered in 212 BC Siracusa in Sicily, the city of Archimedes, a Roman soldier came across this genius contemplating his drew circles on the sand. "Nolite turbare circulos meos!" (Do not break my circles) Archimedes told to the soldier, but this, irritated, stabbed him with the sword, killing him. Today the place of the geometric constructions is on dynamic platforms supported by specialized software. One of these platforms is GeoGebra software. As the inventor stated, GeoGebra is a dynamic mathematics software for all levels of education that joins arithmetic, geometry, algebra and calculus. It offers multiple representations of objects in its graphics, algebra, and spreadsheet views that are all dynamically linked. While other interactive software (e.g. Cabri Geometry, Geometer's Sketchpad) focus on dynamic manipulations of geometrical objects, the idea behind GeoGebra is to connect geometric, algebraic, and numeric representations in an interactive way. You can do constructions with points, vectors, lines, conic sections as well as functions and change them dynamically afterwards. Furthermore, GeoGebra allows you to directly enter and manipulate equations and coordinates. Thus you can easily plot functions, work with sliders to investigate parameters, find symbolic derivatives, and use powerful commands like Root or Sequence, (www.geogebra.org). Because some of us (teachers of different subjects), see the computers in terms of "IT specialist", producers of software and not as a user, we propose this ongoing project in order to bring the computer in the context of approaching successful teaching. We believe that the teacher, regardless of the specialty they teach, shouldn't know what's in the magic box (called generic computer), but he'll have to know that the magic box will help him to assist the student to learn, the route to knowledge becoming pleasant! This paper is an invitation to solve math problems in a natural didactic way using the GeoGebra platform.*

Keywords: Innovation, Geometric Locus, New Way of Learning.

Introduction

There is a struggle to integrate the computer in school. This approach must be read: *"the integration of educational software in education".* This is a related desire to implement new teaching methods in mathematics. An *"educational software application"* is something that everyone could use on a computer, without having advanced knowledge about computers and programming. Draw, build, unite, investigate properties, change shape and size. Properties remain the same? Why? Can you formulate the theorem from this investigation? Prove it rigorously! Experience should not only be lived, but shared. When the action will become more global, this software that I called „The GeoGebra Language" will not only be a working method but also a step in opening a viable way to exchange ideas, and the investigations will become constructions of new methods of investigation of maths phenomena. Different methods of maths teaching have been proposed and kowledge of these methods may help in working out a better teaching strategy. It is not appropriate for a teacher to commit to one particular method. A teacher should adopt a teaching approach after considering the nature of the students, their interests and maturity and the resurse aviable. Every method has certain benefits and few flaws and it is the teacher's work to decide wich method is the best. The investigation above will present an implementation of geogebra in order to solve some maths problems.

Bringing the computer in the context of approaching successful teaching

It is often said more about using interactive methods in teaching mathematics and about their implementation in the curriculum. However we appreciate that the first step must be done when the blackboard and chalk are replaced with dynamic image of mathematical phenomena, integrated in dynamic software like GeoGebra. There are no barriers to this and only the wish to use the system can produce the desired success. If the implementation conformity with the curriculum seems to be difficult, we accept that GeoGebra platform will be a challenge for beginning and maths teachers, if they will accept an innovative method to transmit information, they will encourage students, to spark interest to investigate, to discover the phenomenon mathematically and to justify the results found in rigorous mathematical sense in the end. These were the desires of the new Institute GeoGebra from Romania, to propose to our fellow colleagues a teaching approach, making the invitation to use the GeoGebra platform for dynamic representation of mathematical results. Ultimately, the whole curricula within mathematics could be structured in terms of sequences of GeoGebra using topics with associated learning resources. Students could form teams to explore these sequences, just in the same way they now explore levels within video-game environments, [Gerry Stahl and All, 2010]. We can do constructions with points, vectors, segments, lines, conic sections as well as functions and change them dynamically afterwards. On the other hand, equations and coordinates can be entered directly. Thus, GeoGebra has the ability to deal with variables for numbers, vectors and points, finds derivatives and integrals of functions, [Valerian Antohe, 2009].

Math problems solved with the GeoGebra platform

A step by step construction, which represents the visual interpretation of the mathematical context, a problem of a geometric locus will follows the next steps: constructing geometric figures based on hypotheses, applying geometric transformation, (move the point, move the point along the line, move the line preserving the direction or modify the figure preserving the measure of some angles, etc.). Understanding the relationship between Euclidian construction and proof, we can create demonstration that involves animation and action button, find out geometrically and algebrically connections in a rigorous proof, [Gabriela-Simona Antohe, 2009].

The statement of the first problem: On the fixed line d, are considered the fixed points A and B in the plane and the mobile point M. In the plane are built regular polygons of [AM] and [AN] sides, with m, respectively n number of sides, where m,n∈N, m,n≥3. The circumscribed circles of those polygons are intersected in M and P, (Fig.1). Is required the geometric locus of P, [A. Dafina, 2003].

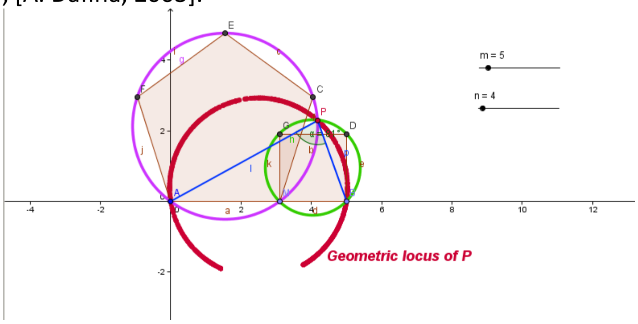

Figure 1: Geometric locus of P for m=5 and n=4

After the investigation with GeoGebra software, some different results could be raised. The geogebra application shows in a real time all the changes and will allow the changing of m and n for different values with the sliders, (Fig.2).

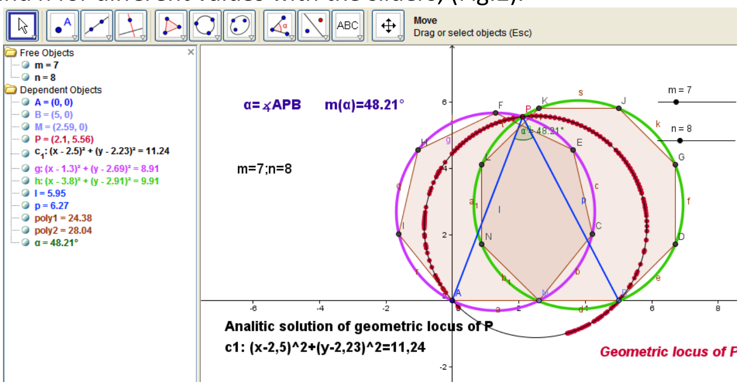

Figure 2: Geometric locus of P for m=5 and n=4

The statement of the second problem: Miquel's Five-Circle Theorem is among a sequence of wonderful theorems in plane geometry bearing his name. Let P1, P2, P3, P4 and P5 be five points. Let Q1=P2P5∩P1P3, Q2=P1P3∩P2P4, Q3=P2P4∩P3P5, Q4=P3P5∩P1P4, and Q5=P1P4∩P2P5. Let the other intersections of the consecutive circumscribed circles of

triangles
Q5Q1P1, Q1Q2P2, Q2Q3P3, Q3Q4P4, and Q4Q5P5 be M1, M2, M3, M4, and M5 respectively. Prove that M1, M2, M3, M4 and M5 are cyclic, (Fig.3).
There are a lot of interesting proofs of this theorem. Miquel's Five-Circle Theorem is difficult to prove algebrically, [Hongbo Li, 2004].

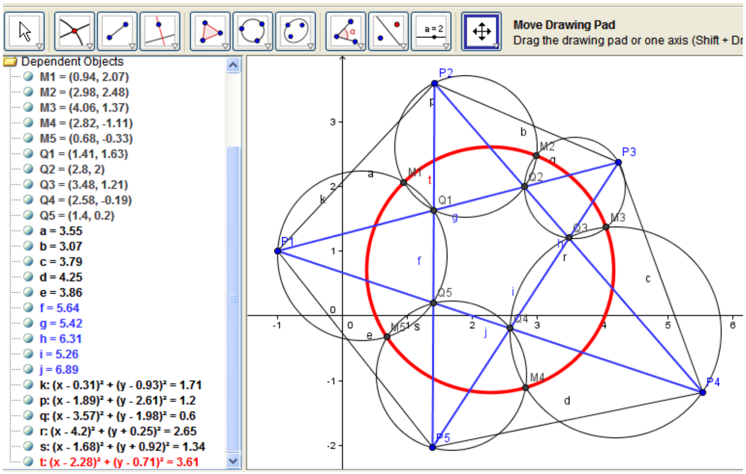

Figure 3: Miquel's Five-Circle Theorem

When n = 3, the three vertices of a triangle are on a unique circle, which can be taken as the unique circle determined by the three edges of the triangle, called the *Miquel 3-circle*, (Fig.4). When n = 4, the 4 edges of a quadrilateral form 4 distinct 3-tuples of edges, each determining a Miquel 3-circle, and Miquel's 4-Circle Theorem says that the 4 Miquel 3-circles pass through a common point (i.e., are concurrent), called the *Miquel 4-point*, (Fig. 5). This combination of perspectives allows the teacher to demonstrate, in front of students and together with them, strategies revealing the "behavior" of figures. Connections between different representations of maths concepts will accomplish here the necessary background for better understanding, steady knowledge of mathematical literature.

One appreciates the pedagogical implications of exploring geometry in a dynamic environment, both as an investigation tool and as a demonstration one, the connection between maths educators and specialists in informatics being one of the best and a challenge at the same time. The term of “Dynamic-Info-Geometry” could be a method of maths teaching and the start of future investigations in applied mathematics, [Gabriela Antohe, 2009].

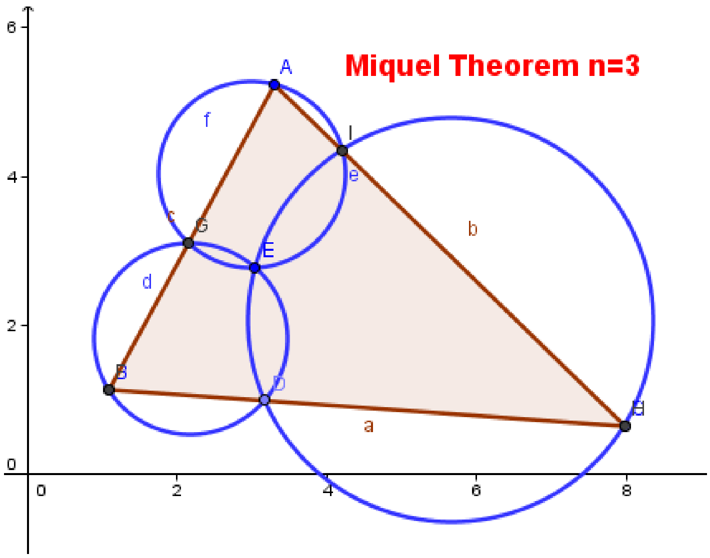

Figure 4: Miquel's 3-Circle Theorem

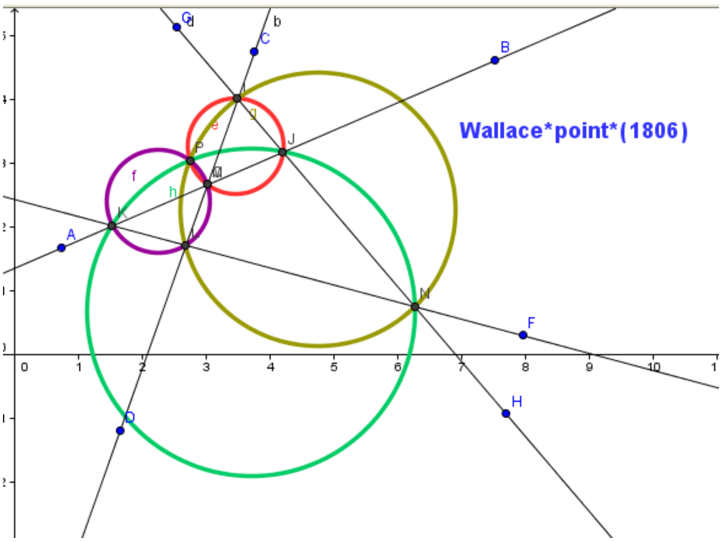

Figure 5: Miquel's 4-Circle Theorem. The position of A, B, F, could be modified but the four circles will have a common point P, the Wallace point

The third problem statement: One of the current problems of education is multidisciplinary treatment of some problems of mathematical modeling. In a study of mathematical modeling of surface water quality I investigated the problem of water quality characteristics of the Danube, such as the evolution of dissolved oxygen concentration. For this we considered the mathematical modeling of these with Spline functions.

The data referring to the dissolved oxygen have been processed in Matlab taking into consideration the determination of a function with the help of spline functions. The same data were processed in geogebra in order to obtain a polynomial function which could describe the data evolution. This example could be a result to the question of the Weierstras problem in order to find the appropriate function which could describe the reality better, find the function and identify methods of interpolation. Weierstrass Law, which shows that any continuous function f can be approached with quite a good precision on a close given interval by a polynomial forms. Unfortunately this theorem does not offer any practical criterion of finding the right polynomial form [**Cline K.S., 2007**].

With GeoGebra, the polynomial form will be:

$P(x)=-0.00001\, x^{11} + 0.0007\, x^{10} - 0.02265\, x^9 + 0.425\, x^8 - 5.10931\, x^7 + 41.1428\, x^6 -$

$225.28496\, x^5 +$

$+833.75065\, x^4 - 2028.84107\, x^3 + 3063.25584\, x^2 - 2551.097\, x + 880.81.$

This form it is obtained by GeoGebra using the command Polynomial [A,B,C,...,K, L],

(Fig. 6).

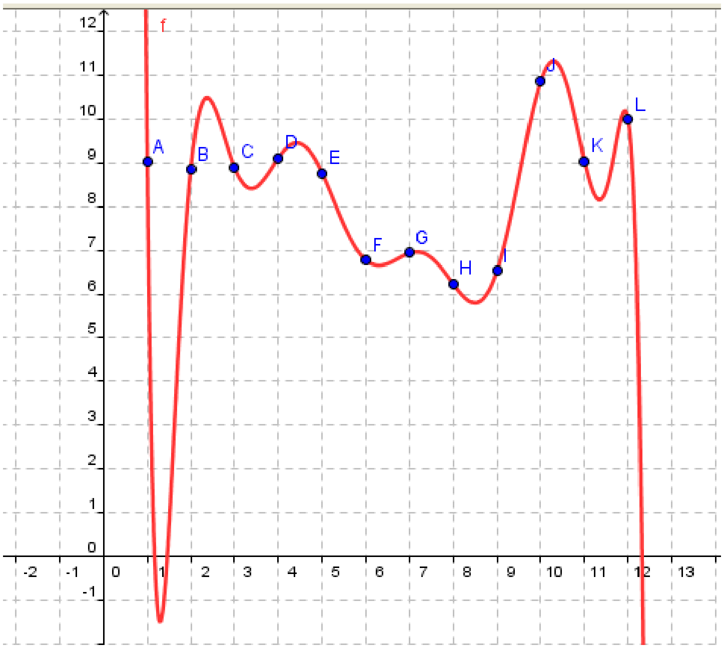

Figure 6: GeoGebra Polynomial form demonstrate that the DO concentration is too small in January 2007

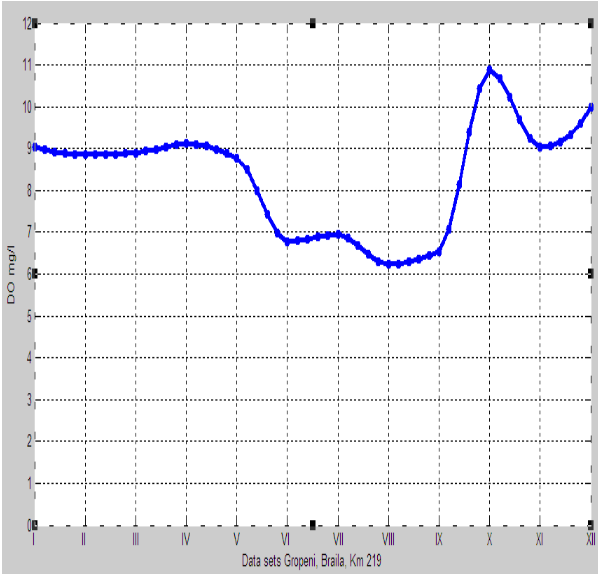

Figure 7: Matlab interpolation with Cubic Spline Function more appropriated to reality.
The program analyzes the Dissolved Oxygen concentration at the Danube River, SGA Braila, Km 219

Analyzing the polynomial function of 11-th degree graph, the great anomaly could be seen during January-February 2007. GeoGebra shows that a polynomial form could not have modelled that reality. This is a convergence to the idea that other function like harmonic function must be analyzed. The interpolation with Spline function, (Fig. 7), was more relevant and the graph represented the hypothesis that the evolution of the quality parameter, [**V.Antohe, C.Stanciu, 2009**].

The fourth problem statement: Let ABC be a triangle and M be an inner point of the triangle so that AM=BC. Shoe that max{(BM/AC);(CM/AB)}≥$2^{1/2}$-1 and this is the best possible constant.

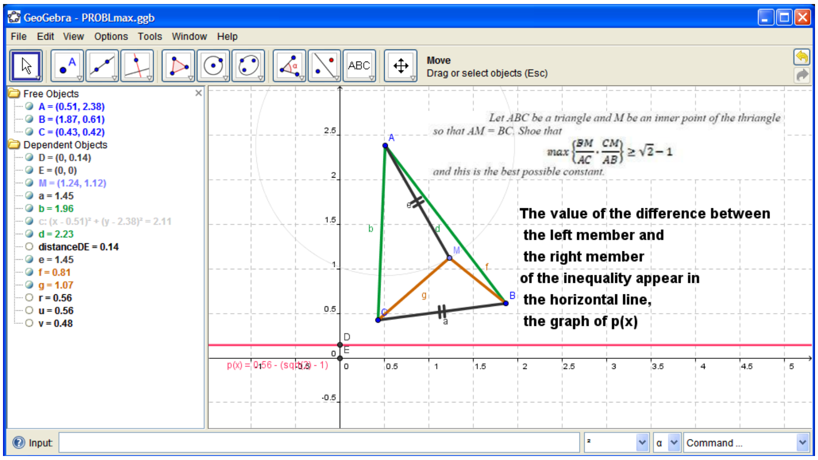

Figure 8: The problem of a geometric inequality relation

After the geometrical context was done, (Fig.8), the values of the left and on the right hand were analyzed. The value of the difference between the left member and the right member of the inequality appear in the horizontal line, the graph of p(x). Even if this modelling does not give a demonstration on GeoGebra rigorous analysis of the successive positions of the peaks of the triangle, keeping the requirements of the problem will emphasize such inequality in borderline cases, cases that can get out of context. Of course for educated this exercise of geometric and algebra realization of context is a good exercise, a first step to understanding, analyzing and demonstrating the problem rigorously. Other two similar problems are presented in these imagines, (Fig. 8).

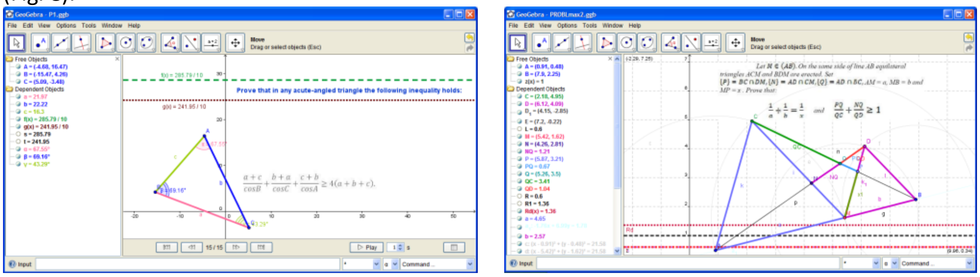

Figure 8: Other problems of a geometric inequality relation

Conclusion

GeoGebra provides good opportunity for students to work in pairs and talk through the project together. Attractive presentations prepared in advance, not only capture students' attention but also may lessen the immediate cognitive load for educated and educators. In addition to what is traditionally recognized as benefits, a lot of teachers often use real world models. In order to enhance the image mathematics by creating a "halo effect", the proposed efficient space for this will be the GeoGebra platform. The teachers who use GeoGebra must be more specific, more "open minded", willing to allow for experimentation, and give more guidance at the start of any geogebra experiment. Dynamic geometry offers opportunities to bring the real world into the classroom, adding visualization, color and animation. This would not be possible in a traditional classroom. This GeoGebra thinking is expected in various topics of the curriculum but, if they are not found there, we shall connect the GeoGebra thinking with topics and other different experiences, in a model of more efficient curricula.

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