Exploring Medial Triangle using Geogebra

Sanjay Gulati
Shri Shankaracharya Vidyalaya , Hudco , Bhilai (Chhattisgarh)
India
http://mathematicsbhilai.blogspot.com
sanjaybhil@gmail.com

ABSTRACT. *In this paper, we will explore and visualize geometry of medial triangle. We will explore how geometry of a medial triangle is connected with the original triangle.*

1. Introduction

The triangle formed by joining the midpoints of the sides of a given triangle is called the medial triangle.
Triangle ABC is a given triangle; with D, E and F are respectively the mid points of sides AB, BC and CA.

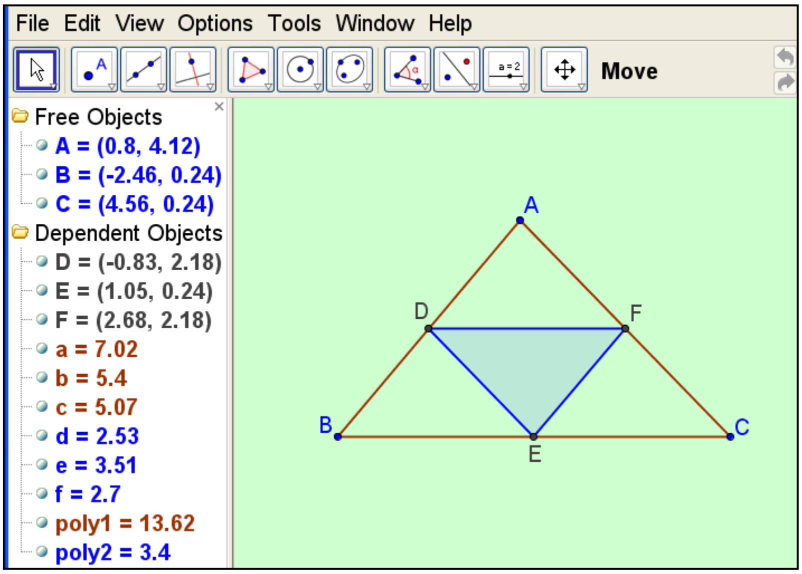

Fig.1. Triangle DEF it is called the medial triangle of a given triangle ABC.

We know that a line segment joining midpoints of two sides of a triangle is parallel to the third side and half in length of third side.

So, in triangle ABC, DE , DF and FD are respectively are parallel to sides AC , AB and BC.

We also note that area of four triangles ADF, BDE, EFC and DEF are equal and is equal to one fourth of the area of triangle ABC.

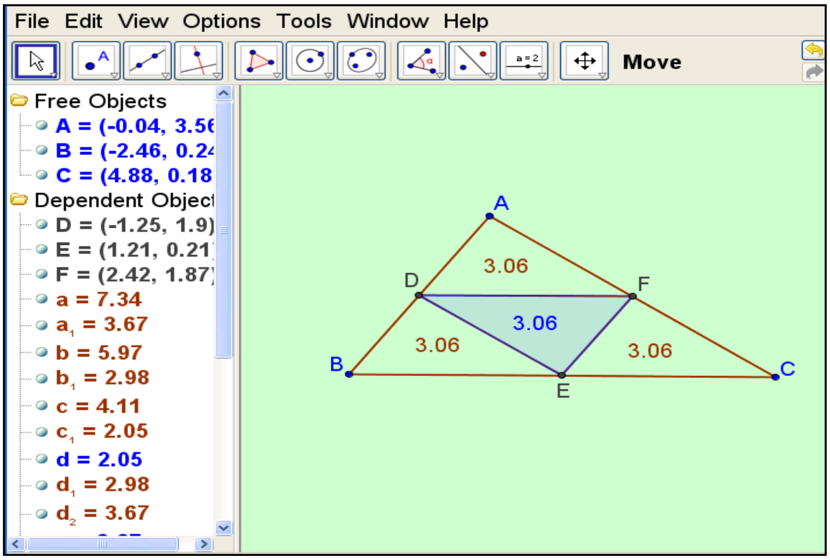

Fig.2. Thus area of medial triangle DEF is $\frac{1}{4}$ th of the area of given triangle ABC.

2. Centres in a medial triangle

a) Centroid of Medial Triangle

Consider the figure give below:

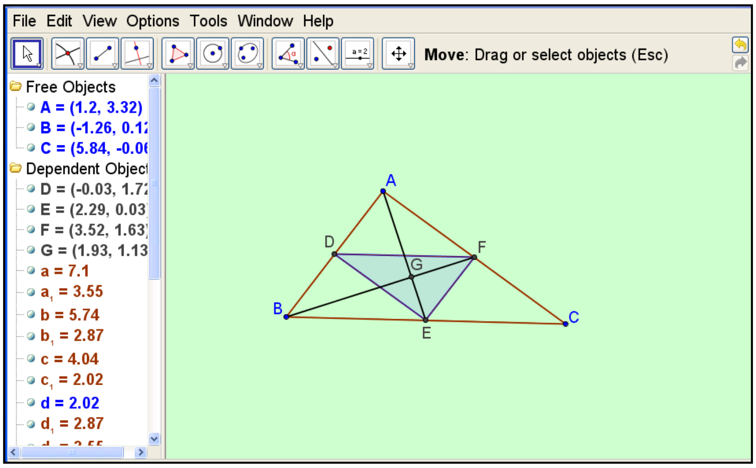

Fig.3. Centroid G

We see that ADEF is a parallelogram (AD || EF and AF || DE), hence segments DF and AE bisect each other (diagonals of a parallelogram bisect each other). Similarly DE and FB also bisect each other. Thus the medians of triangle DEF lie along the medians of triangle ABC. So both triangles ABC and DEF have same centroid (G).

b) Orthocentre of Medial Triangle

Draw altitudes of triangle DEF from vertices D and E (see figure below) , these altitudes are perpendicular bisectors of triangle ABC (DF || BC and EI is perpendicular to DF, so EI is perpendicular to BC. Also E is the midpoint of BC; hence EI is perpendicular bisector of BC.

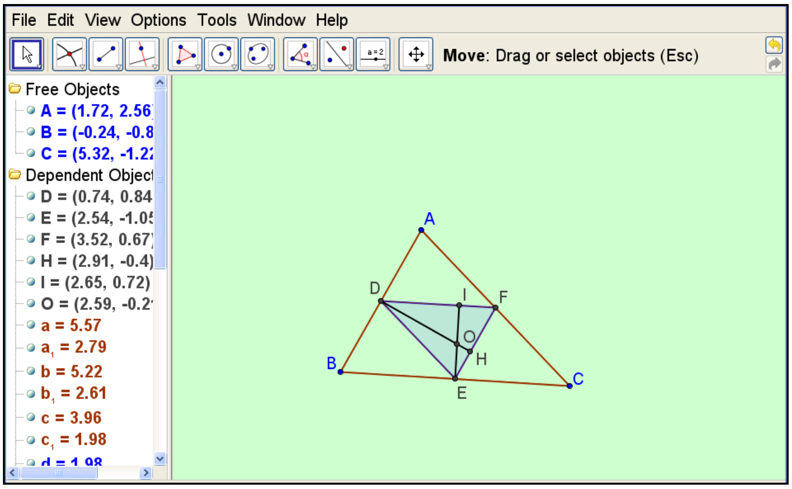

Fig.4. These altitudes intersect at O , the circumcentre of triangle ABC , thus otrhocentre of triangle DEF coincides with the circumcentre of triangle ABC.

Circum Circle of Medial Triangle

Let us now explore about the circum circle of the medial triangle DEF. Let us denote circum circle of triangle DEF by a symbol C(DEF).

In the following figure, AA_1 is the altitude of triangle ABC from vertex A.

We have EF || AB and EF = $\frac{1}{2}$AB (1)

In triangle ABA_1 ,

$\angle AA_1B = 90°$ and D is the midpoint of AB;

Thus, $A_1D = \frac{1}{2}AB$ (2)

[Segment joining midpoint of hypotenuse to vertex containing 90° is half the length of hypotenuse] thus $A_1D = EF$ [from 1 and 2]

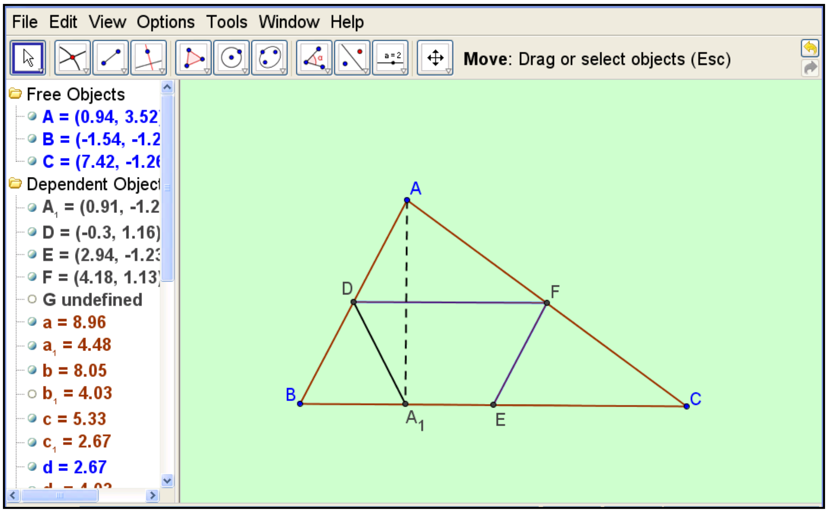

Fig.5. Medial triangle

Thus A_1DFE is an isosceles trapezium and hence it is con-cyclic. It follows from here that point A_1 lies on the circum circle of triangle DEF.

Similarly, if we draw altitudes from vertices B and C, then their respective feet B_1 and C_1 also lie on C(DEF) , see figure below.

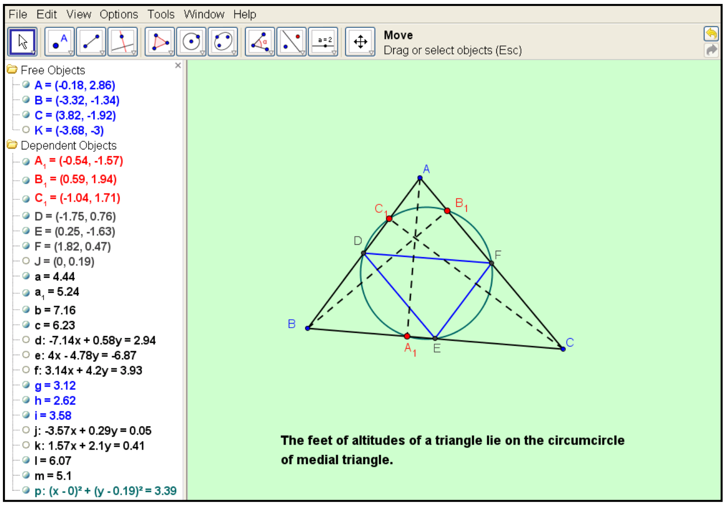

Fig. 6. The circum circle of triangle DEF

Thus we can say that "The feet of the altitudes of a triangle ABC lie on the circum circle of its medial triangle DEF."

Let us explore further.

In the following figure, H is the orthocenter of triangle ABC, let A_2 is the midpoint of the line segment joining vertex A to the orthocenter H.

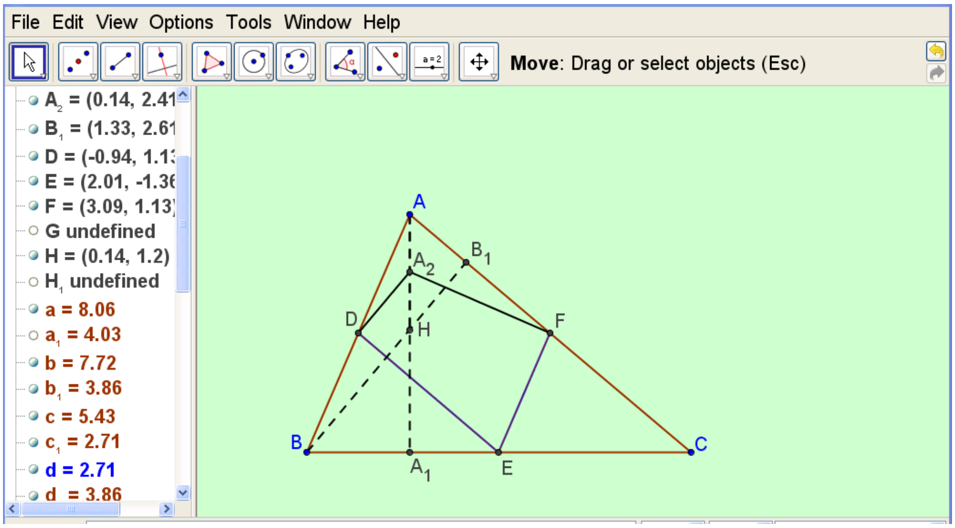

Fig. 7. The orthocenter of triangle ABC

DE || AC and DA_2 || BH [Line segment joining midpoints of two sides of a triangle is parallel to the third side.]

But, BH $\perp$ AC [BB_1 is altitude to AC]

So, DE $\perp$ DA_2

Similarly, EF || AB and A_2F || HC

But HC $\perp$ AB, so EF $\perp$ HC

From the above it follows that $DEFA_2$ is a cyclic quadrilateral and point A_2 lies on the circum circle of triangle DEF.

In the same way if we locate B_2 and C_2 , the mid points of HB and HC respectively , we find that these points also lies on the circum circle of triangle DEF.

Thus we can say that "the three mid points of the line segments joining the orthocenter to vertices of the triangle lie on the circum circle of the medial triangle."

From the above discussions , we have 9 points , D,E,F , A_1 , B_1 , C_1 , A_2 , B_2 , C_2 , all lying on the circum circle of the medial triangle i.e. these 9 points are con cyclic , this circle is called the **Nine Point Circle of triangle ABC**.

Since $DEFA_2$ it is cyclic with $A_2D \perp DE$ and $A_2F \perp EF$, then A_2E is a diameter of the nine point circle. The center N of the nine point circle will be the midpoint of the diameter A_2E as shown in the following figure.

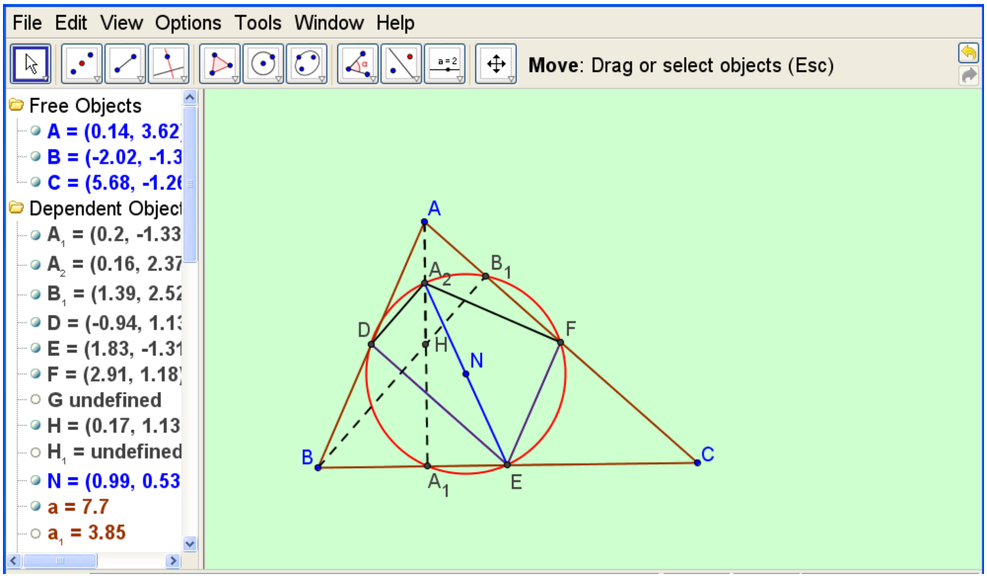

Fig. 8. Showing center of Nine point circle passing through D , E , F and A_2

References:

1. www.irmo.ie
2. http://jwilson.coe.uga.edu/EMAT6680Fa05/Ersoz/Assignment%204/medialtriangle.html

Return to Issue Vol2.No.2 →